DYNAMICAL SYSTEMS ACCEPTING THE NORMAL SHIFT.

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Newtonian dynamical system on a Riemannian manifold M is a system of ordinary differential equations of the form

$$\dot{x}^i = v^i \qquad\qquad \nabla_t v^i = F^i(\mathbf{x},\mathbf{v})$$

describing the motion of a mass point with the unit mass in the force field $\mathbf{F}$ on M. Let S be the hypersurface in M and let $\mathbf{n}(P)$ be the unit normal vector to S at the point P. Taking the initial velocity of unit mass points on S for $t=0$ as $\mathbf{v} = v(P)\mathbf{n}(P)$ we define the shift of S along the trajectories of the dynamical system: $f_t : S \longrightarrow S_t$. The transformation f_t is called the normal shift if the trajectories of the dynamical system cross the hypersurfaces S_t along their normal vectors for any value of t.

Definition. Newtonian dynamical system on M is called the system accepting the normal shift if for any hypersurface S one can find the function $v(P)$ on S for the modulus of initial velocity defining the normal shift of S. That Newtonian system is called the strongly normal dynamical system if the above function $v(P)$ can be normalized by the condition $v(P_0) = v_0$ for any choice of $P_0 \in S$ and for any nonzero $v_0 \in \mathbb{R}$.

Theorem. *The strong normality condition for the Newtonian dynamical system is equivalent to the following system of partial differential equations for its force field* $\mathbf{F}(\mathbf{x},\mathbf{v})$

$$\begin{cases} (v^{-1}F_i + \tilde{\nabla}_i(F^k N_k))P^i_q = 0 \\ (\nabla_i F_k + \nabla_k F_i - 2v^{-2}F_i F_k)N^k P^i_q + v^{-1}(\tilde{\nabla}_k F_i F^k - \tilde{\nabla}_k F^r N^k N_r F_i)P^i_q = 0 \\ P^k_i P^q_j \left(N^r \dfrac{\tilde{\nabla}_r F_k}{v} F_q - \nabla_q F_k \right) = P^k_i P^q_j \left(N^r \dfrac{\tilde{\nabla}_r F_q}{v} F_k - \nabla_k F_q \right) \\ P^k_i \tilde{\nabla}_k F^q P^j_q = \dfrac{P^k_r \tilde{\nabla}_k F^q P^r_q}{n-1} P^j_i \end{cases}$$

where $N^i = |\mathbf{v}|^{-1}v^i$, $P^i_k = \delta^i_k - N_k N^i$ *and the covariant derivatives are defined as* $\nabla_i F^k = \partial F^k/\partial x^i + \Gamma^k_{ij}F^j - \partial F^k/\partial v^s \Gamma^s_{ij} v^j$, $\tilde{\nabla}_i F^k = \partial F^k/\partial v^i$.

The concept of dynamical systems accepting the normal shift was introduced in [1] and [2] (see also [3]). Multidimensionlal dynamical systems accepting the

normal shift are considered in [4] (see also [5]). The above results for the dynamical systems on Riemannian manifolds are published in [6] and [7]. Their generalization for higher order (non-Newtonian) dynamical systems is considered in [8].

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