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THE ELECTRO-WEAK AND COLOR BUNDLES FOR THE STANDARD MODEL IN A GRAVITATION FIELD.

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Abstract. It is known that the Standard Model describing all of the currently known elementary particles is based on the $\mathrm{U}(1)\times\mathrm{SU}(2)\times\mathrm{SU}(3)$ symmetry. In order to implement this symmetry on the ground of a non-flat space-time manifold one should introduce three special bundles. Some aspects of the mathematical theory of these bundles are studied in this paper.

1. The Dirac bundle and its basic fields.

Spin is a common property of all elementary particles. It is described in terms of the Dirac bundle DM, where M is a space-time manifold. The Dirac bundle is a four-dimensional complex bundle over the four-dimensional real manifold M. The detailed description of this bundle can be found in [1], [2], and [3]. In this section we shall remind in brief some of its properties.

The base manifold M of the Dirac bundle DM is equipped with the following three geometric structures which are well-known in General Relativity:

(1) a pseudo-Euclidean Minkowski-type metric $\mathbf{g}$;
(2) an orientation;
(3) a polarization

(see details in [4]). The Dirac bundle DM is linked to the above three structures through frame pairs. The most popular frame pairs are listed in the following table:

Canonically orthonormal chiral frames	→	Positively polarized right orthonormal frames
P-reverse anti-chiral frames	→	Positively polarized left orthonormal frames
T-reverse anti-chiral frames	→	Negatively polarized right orthonormal frames
PT-reverse chiral frames	→	Negatively polarized left orthonormal frames

(1.1)

(see details in [1] and [2]). Each of the frames in the right hand side of the table (1.1) is an ordered set of four vector fields $\boldsymbol{\Upsilon}_0$, $\boldsymbol{\Upsilon}_1$, $\boldsymbol{\Upsilon}_2$, $\boldsymbol{\Upsilon}_3$ linearly independent

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at each point p of some open domain $U \subset M$. We denote it $(U,\, \mathbf{\Upsilon}_0,\, \mathbf{\Upsilon}_1,\, \mathbf{\Upsilon}_2,\, \mathbf{\Upsilon}_3)$. Similarly, each of the frames listed in the left hand side of the table (1.1) is an ordered set of four spinor fields $\mathbf{\Psi}_1$, $\mathbf{\Psi}_2$, $\mathbf{\Psi}_3$, $\mathbf{\Psi}_4$, i. e. four smooth sections of the Dirac bundle DM, linearly independent at each point $p \in U$. We denote it $(U,\, \mathbf{\Psi}_1,\, \mathbf{\Psi}_2,\, \mathbf{\Psi}_3,\, \mathbf{\Psi}_4)$. The spinor frames in the left hand side of the table (1.1) are defined with the use of the following basic spin-tensorial fields (see [2]):

Symbol	Name	Spin-tensorial type
$\mathbf{g}$	Metric tensor	$(0,0\vert 0,0\vert 0,2)$
$\mathbf{d}$	Skew-symmetric metric tensor	$(0,2\vert 0,0\vert 0,0)$
$\mathbf{H}$	Chirality operator	$(1,1\vert 0,0\vert 0,0)$
$\mathbf{D}$	Dirac form	$(0,1\vert 0,1\vert 0,0)$
$\boldsymbol{\gamma}$	Dirac γ-field	$(1,1\vert 0,0\vert 1,0)$

(1.2)

Note that the metric tensor $\mathbf{g}$ inherited from M is interpreted as a spin-tensorial field of DM in the table (1.2).

2. The electro-weak bundles ɄM and SɄM.

The first bundle $ɄM$ of the Standard Model is a one-dimensional complex bundle over the space-time manifold M. It is equipped with a Hermitian scalar product

$$Đ(\mathbf{X},\mathbf{Y}) = Đ_{11}\,\overline{X^1}\,Y^1 = \overline{Đ(\mathbf{Y},\mathbf{X})}. \tag{2.1}$$

(compare (2.1) with the formula (5.18) in [2]). Here X^1 and Y^1 are the components of two vector fields of the bundle $ɄM$ in some frame $(U,\mathbf{\Psi}_1)$. Any frame of $ɄM$ is a smooth section of this bundle nonzero at each point of some open domain $U \subset M$.

Tensor fields for $ɄM$ are defined in a way similar to those for the Dirac bundle. In addition to $ɄM$, we construct its conjugate and Hermitian conjugate bundles:

$$Ʉ^*M, \qquad Ʉ^\dagger M, \qquad Ʉ^{*\dagger}M = Ʉ^{\dagger *}M. \tag{2.2}$$

Then, using (2.2), we define the following tensor products:

$$Ʉ^\varepsilon_\eta M = \overbrace{ɄM \otimes \ldots \otimes ɄM}^{\varepsilon\text{ times}} \otimes \underbrace{Ʉ^*M \otimes \ldots \otimes Ʉ^*M}_{\eta\text{ times}}, \tag{2.3}$$

$$\bar{Ʉ}^\sigma_\zeta M = \overbrace{Ʉ^{\dagger *}M \otimes \ldots \otimes Ʉ^{\dagger *}M}^{\sigma\text{ times}} \otimes \underbrace{Ʉ^\dagger M) \otimes \ldots \otimes Ʉ^\dagger M}_{\zeta\text{ times}}. \tag{2.4}$$

The tensor product of the above two tensor bundles (2.3) and (2.4) is denoted as

$$Ʉ^\varepsilon_\eta \bar{Ʉ}^\sigma_\zeta M = Ʉ^\varepsilon_\eta M \otimes \bar{Ʉ}^\sigma_\zeta M. \tag{2.5}$$

Due to (2.5) the tensorial type of a UM-tensor is given by four numbers $(\varepsilon, \eta|\sigma, \zeta)$. In particular, the Hermitian scalar product (2.1) is given by a tensor field $\mathbf{D}$ of the type $(0,1|0,1)$. This field could be upended to those in the table (1.2).

Definition 2.1. A frame $(U, \boldsymbol{\Psi}_1)$ of the electro-weak bundle UM is called an *orthonormal frame* with respect to $\mathbf{D}$ if $D_{11} = 1$ in this frame.

Assume that $(U, \boldsymbol{\Psi}_1)$ and $(\tilde{U}, \tilde{\boldsymbol{\Psi}}_1)$ are two orthonormal frames with respect to $\mathbf{D}$ and assume that their domains are overlapping: $U \cap \tilde{U} \neq \varnothing$. Then

$$\tilde{\boldsymbol{\Psi}}_1 = \mathfrak{S}^1_1\,\boldsymbol{\Psi}_1, \text{ where } |\mathfrak{S}^1_1| = 1. \tag{2.6}$$

The coefficient $\mathfrak{S}^1_1$ in (2.6) is interpreted as a unitary matrix $\mathfrak{S} \in \mathrm{U}(1)$.

The electro-weak bundle UM is a complex bundle over the real manifold M. For this reason it is equipped with the involution of complex conjugation τ:

$$U^{\varepsilon}_{\eta}\bar{U}^{\sigma}_{\zeta}M \;\underset{\tau}{\overset{\tau}{\rightleftarrows}}\; U^{\sigma}_{\zeta}\bar{U}^{\varepsilon}_{\eta}M. \tag{2.7}$$

The same is true for the Dirac bundle DM (see the formula (1.2) in [3]).

The second electro-weak bundle SUM is a little bit more complicated. It is a two-dimensional complex bundle over the space-time manifold M. Like UM, this bundle is equipped with a Hermitian scalar product

$$D(\mathbf{X}, \mathbf{Y}) = \sum_{i=1}^{2}\sum_{\bar{j}=1}^{2} D_{i\bar{j}}\,\overline{X^{\bar{j}}}\,Y^i = \overline{D(\mathbf{Y}, \mathbf{X})}. \tag{2.8}$$

Tensor bundles associated with SUM are introduced by the following formulas:

$$SU^{\pi}_{\rho}M = \overbrace{SUM \otimes \ldots \otimes SUM}^{\pi\text{ times}} \otimes \underbrace{SU^{*}M \otimes \ldots \otimes SU^{*}M}_{\rho\text{ times}}, \tag{2.9}$$

$$\overline{SU}^{\omega}_{\mu}M = \overbrace{SU^{\dagger *}M \otimes \ldots \otimes SU^{\dagger *}M}^{\omega\text{ times}} \otimes \underbrace{SU^{\dagger}M) \otimes \ldots \otimes SU^{\dagger}M}_{\mu\text{ times}}. \tag{2.10}$$

$$SU^{\pi}_{\rho}\overline{SU}^{\omega}_{\mu}M = SU^{\pi}_{\rho}M \otimes \overline{SU}^{\omega}_{\mu}M. \tag{2.11}$$

The formulas (2.9), (2.10), and (2.11) are analogous to (2.3), (2.4), and (2.5). As we see in (2.11), the tensorial type of a SUM-tensor is determined by four numbers $(\pi, \rho|\omega, \mu)$. Like (2.1), the Hermitian scalar product (2.8) is determined by a tensor-field $\mathbf{D}$ of the type $(0,1|0,1)$. We call it the *Hermitian metric tensor* of the electro-weak bundle SUM.

Apart from $\mathbf{D}$, there is another basic tensor field associated with the bundle SUM. It is a skew-symmetric tensor of the type $(0,2|0,0)$. We denote it by $\mathbf{d}$ and call the *skew-symmetric metric tensor*. This tensor is assumed to be nonzero at each point p of the space-time manifold M. Therefore one can introduce the *dual skew-symmetric metric tensor*. By tradition we denote it with the same symbol

$\not\mathbf{d}$. The components $\not d^{ij}$ of the dual skew-symmetric metric tensor form the matrix inverse to the matrix formed by the components $\not d_{ij}$ of the initial tensor $\not\mathbf{d}$:

$$\sum_{j=1}^{2} \not d^{ij}\, \not d_{jk} = \delta^i_k, \qquad\qquad \sum_{j=1}^{2} \not d_{ij}\, \not d^{jk} = \delta^k_i. \tag{2.12}$$

The mutually inverse matrices $\not d_{ij}$ and $\not d^{ij}$ from (2.12) are used in index raising and index lowering procedures for $S\not UM$-tensors. Let's apply them to the tensor $\not\mathbf{D}$:

$$\not D^{i\bar{j}} = \sum_{p=1}^{2}\sum_{\bar{q}=1}^{2} \not d^{ip}\, \overline{\not d^{\bar{j}\bar{q}}}\, \not D_{p\bar{q}}. \tag{2.13}$$

Definition 2.2. The skew-symmetric metric tensor $\not\mathbf{d}$ is called *concordant with the Hermitian scalar product* (2.8) if the matrix (2.13) is inverse to the matrix $\not D_{i\bar{j}}$ in the sense of the following equalities:

$$\sum_{\bar{a}=1}^{2} \not D^{i\bar{a}}\, \not D_{j\bar{a}} = \delta^i_j, \qquad\qquad \sum_{a=1}^{2} \not D_{a\bar{j}}\, \not D^{a\bar{i}} = \delta^{\bar{i}}_{\bar{j}}. \tag{2.14}$$

Note that almost the same relationships are valid for the spin-tensorial fields $\mathbf{d}$ and $\mathbf{D}$ associated with the Dirac bundle (see formulas (6.24) and (6.25) in [2]).

Definition 2.3. A frame $(U,\, \mathbf{\Psi}_1,\, \mathbf{\Psi}_2)$ of the bundle $S\not UM$ is called an *orthonormal frame* with respect to $\not\mathbf{D}$ if $\not\mathbf{D}$ is given by the unit matrix in this frame:

$$\not D_{i\bar{j}} = \begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix}. \tag{2.15}$$

Definition 2.4. A frame $(U,\, \mathbf{\Psi}_1,\, \mathbf{\Psi}_2)$ of the bundle $S\not UM$ is called an *orthonormal frame* with respect to $\not\mathbf{d}$ if $\not\mathbf{d}$ is given by the following matrix in this frame:

$$\not d_{ij} = \begin{Vmatrix} 0 & 1 \\ -1 & 0 \end{Vmatrix}. \tag{2.16}$$

Definition 2.5. A frame $(U,\, \mathbf{\Psi}_1,\, \mathbf{\Psi}_2)$ of the electro-weak bundle $S\not UM$ is called an *orthonormal frame* if it is orthonormal with respect to $\not\mathbf{D}$ and $\not\mathbf{d}$ simultaneously, i. e. if both equalities (2.15) and (2.16) are valid in this frame.

Theorem 2.1. *The tensor fields $\not\mathbf{D}$ and $\not\mathbf{d}$ associated with the electro-weak bundle $S\not UM$ are concordant in the sense of the definition 2.2 if and only if for each point $p \in M$ there is an orthonormal frame $(U,\, \mathbf{\Psi}_1,\, \mathbf{\Psi}_2)$ in some neighborhood U of p.*

The proof of this theorem is left to the reader. It is rather simple. Note that the concordance of $\mathbf{d}$ and $\mathbf{D}$ in the case of the Dirac bundle DM follows from the definition of these spin-tensorial fields. In the present case it is introduced as an additional requirement for the fields $\not\mathbf{D}$ and $\not\mathbf{d}$ and for the bundle $S\not UM$ itself.

Assume that we have two orthonormal frames $(U,\, \mathbf{\Psi}_1,\, \mathbf{\Psi}_2)$ and $(\tilde{U},\, \tilde{\mathbf{\Psi}}_1,\, \tilde{\mathbf{\Psi}}_2)$ of the bundle SUM with overlapping domains $U \cap \tilde{U} \neq \varnothing$. Then

$$\tilde{\mathbf{\Psi}}_i = \sum_{j=1}^{2} \mathfrak{S}^j_i\, \mathbf{\Psi}_j \tag{2.17}$$

at each point $p \in U \cap \tilde{U}$. The transition matrix $\mathfrak{S}$ with the components $\mathfrak{S}^j_i$ in (2.17) is a unitary matrix with $\det \mathfrak{S} = 1$, i.e. $\mathfrak{S} \in \mathrm{SU}(2)$.

Like UM, the second electro-weak bundle SUM is equipped with the involution of complex conjugation. We denote it with the same symbol τ:

$$SU^{\pi}_{\rho}\overline{SU}^{\omega}_{\mu}M \ \underset{\tau}{\overset{\tau}{\rightleftarrows}}\ SU^{\omega}_{\mu}\overline{SU}^{\pi}_{\rho}M. \tag{2.18}$$

The diagram (2.18) is analogous to (2.7) and to the diagram (1.2) in [3].

3. The color bundle SUM.

The color bundle SUM is used to describe the color states of quarks (see [5]). This is a three-dimensional complex bundle over the real space-time manifold M. For this reason it is equipped with the involution of complex conjugation τ:

$$SU^{\varkappa}_{\lambda}\overline{SU}^{\varsigma}_{\chi}M \ \underset{\tau}{\overset{\tau}{\rightleftarrows}}\ SU^{\varsigma}_{\chi}\overline{SU}^{\varkappa}_{\lambda}M. \tag{3.1}$$

The tensor bundles $SU^{\varkappa}_{\lambda}\overline{SU}^{\varsigma}_{\chi}M$ and $SU^{\varsigma}_{\chi}\overline{SU}^{\varkappa}_{\lambda}M$ in (3.1) are defined as follows:

$$SU^{\varkappa}_{\lambda}M = \overbrace{SUM \otimes \ldots \otimes SUM}^{\varkappa\text{ times}} \otimes \underbrace{SU^{*}M \otimes \ldots \otimes SU^{*}M}_{\lambda\text{ times}}, \tag{3.2}$$

$$\overline{SU}^{\varsigma}_{\chi}M = \overbrace{SU^{\dagger *}M \otimes \ldots \otimes SU^{\dagger *}M}^{\varsigma\text{ times}} \otimes \underbrace{SU^{\dagger}M) \otimes \ldots \otimes SU^{\dagger}M}_{\chi\text{ times}}. \tag{3.3}$$

$$SU^{\varkappa}_{\lambda}\overline{SU}^{\varsigma}_{\chi}M = SU^{\varkappa}_{\lambda}M \otimes \overline{SU}^{\varsigma}_{\chi}M. \tag{3.4}$$

The formulas (3.2), (3.3), (3.4) are analogous to (2.3), (2.4), (2.5) and (2.9), (2.10), (2.11). In addition to τ, the color bundle SUM is equipped with two tensorial fields $\mathbf{D}$ and $\mathbf{d}$. The tensorial field $\mathbf{D}$ is introduced through the Hermitian scalar product

$$D(\mathbf{X}, \mathbf{Y}) = \sum_{i=1}^{2}\sum_{\bar{j}=1}^{2} D_{i\bar{j}}\, \overline{X^{\bar{j}}}\, Y^i = \overline{D(\mathbf{Y}, \mathbf{X})}. \tag{3.5}$$

It is a field of the type $(0,1|0,1)$. The second tensorial field $\mathbf{d}$ is a nonzero completely skew-symmetric field of the type $(0,3|0,0)$. For its components we have:

$$d_{ijk} = -d_{jik}, \qquad d_{ijk} = -d_{ikj}, \qquad d_{ijk} = -d_{kji}. \tag{3.6}$$

The formulas (3.6) means that all of the components of the tensor $\mathbf{d}$ can be expressed through d_{123}. The following definition is based on this feature.

Definition 3.1. A frame $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2,\,\mathbf{\Psi}_3)$ of the bundle SUM is called an *orthonormal frame* with respect to $\mathbf{d}$ if $d_{123}=1$ in this frame.

Definition 3.2. A frame $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2,\,\mathbf{\Psi}_3)$ of the bundle SUM is called an *orthonormal frame* with respect to $\mathbf{D}$ if $\mathbf{D}$ is given by the unit matrix in this frame:

$$D_{i\bar{j}}=\begin{Vmatrix}1&0&0\\0&1&0\\0&0&1\end{Vmatrix}.$$

Let $\mathbf{b}$ be a completely skew-symmetric tensor field of the type $(3,0|0,0)$ associated with the color bundle SUM. Then we have the identities similar to (3.6):

$$b^{ijk}=-b^{jik},\qquad b^{ijk}=-b^{ikj},\qquad b^{ijk}=-b^{kji}.\tag{3.7}$$

The identities (3.7) mean that the components of the tensor $\mathbf{b}$ are completely determined if the component b^{123} is fixed.

Definition 3.3. A completely skew-symmetric tensor $\mathbf{b}$ of the type $(3,0|0,0)$ is called *inverse to the tensor* $\mathbf{d}$ if $d_{123}\cdot b^{123}=1$ for any frame $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2,\,\mathbf{\Psi}_3)$.

For the sake of economy the tensor $\mathbf{b}$ inverse to the tensor $\mathbf{d}$ is denoted by the same symbol, i. e. we write $\mathbf{b}=\mathbf{d}$. This usage of symbols makes no confusion since the initial and the inverse tensors are of different types $(0,3|0,0)$ and $(3,0|0,0)$ respectively.

Definition 3.4. The completely skew-symmetric tensor field $\mathbf{d}$ is called *concordant with the Hermitian scalar product* (3.5) if the following identity is fulfilled:

$$\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}d^{ijk}\,D_{i\bar{i}}\,D_{j\bar{j}}\,D_{k\bar{k}}=\overline{d_{\bar{i}\bar{j}\bar{k}}}.\tag{3.8}$$

Note that the equalities (2.13) and (2.14) can be transformed to the following equality for the components of the electro-weak tensors $\mathbf{D}$ and $\mathbf{d}$:

$$\sum_{i=1}^{2}\sum_{j=1}^{2}d^{ij}\,D_{i\bar{i}}\,D_{j\bar{j}}=\overline{d_{\bar{i}\bar{j}}}.\tag{3.9}$$

The equality (3.9) is a two-dimensional analog of the equality (3.8). Therefore, the definition 2.2 also is a two-dimensional analog of the definition 3.4.

Definition 3.5. A frame $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2,\,\mathbf{\Psi}_3)$ of the color bundle SUM is called an *orthonormal frame* if it is orthonormal with respect to $\mathbf{D}$ and $\mathbf{d}$ simultaneously.

Theorem 3.1. *The tensor fields* $\mathbf{D}$ *and* $\mathbf{d}$ *associated with the color bundle* SUM *are concordant in the sense of the definition 3.4 if and only if for each point* $p\in M$ *there is an orthonormal frame* $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2,\,\mathbf{\Psi}_3)$ *in some neighborhood* U *of* p.

This theorem 3.1 is similar to the theorem 2.1. Its proof is also left to the reader.

Assume that we have two orthonormal frames of the color bundle SUM. Let's denote them $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2,\,\mathbf{\Psi}_3)$ and $(\tilde{U},\,\tilde{\mathbf{\Psi}}_1,\,\tilde{\mathbf{\Psi}}_2,\,\tilde{\mathbf{\Psi}}_3)$. Assume that their domains

U and $\tilde{U}$ are overlapping, i. e. $U \cap \tilde{U} \neq \varnothing$. Then at each point $p \in U \cap \tilde{U}$ we have the transition formula relating two frames $(U,\, \mathbf{\Psi}_1,\, \mathbf{\Psi}_2,\, \mathbf{\Psi}_3)$ and $(\tilde{U},\, \tilde{\mathbf{\Psi}}_1,\, \tilde{\mathbf{\Psi}}_2,\, \tilde{\mathbf{\Psi}}_3)$:

$$\tilde{\mathbf{\Psi}}_i = \sum_{j=1}^{3} \mathfrak{S}_i^j\, \mathbf{\Psi}_j. \tag{3.10}$$

The transition matrix $\mathfrak{S}$ with the components $\mathfrak{S}_i^j$ in (3.10) is a unitary matrix with $\det \mathfrak{S} = 1$, i. e. it is an element of the special unitary group: $\mathfrak{S} \in \mathrm{SU}(3)$.

Conclusion. As we see now, the structural groups for all of the above bundles are revealed through the transition matrices relating orthonormal frames, while orthonormal frames themselves are defined with the use of some special tensorial fields associated with these bundles.

4. Leptons and quarks.

Basic elementary particles in the Standard Model are subdivided into two groups, the first group is formed by leptons and the second group is formed by quarks. Other particles arise as quanta of gauge fields. Leptons are described by the $\mathrm{U}(1) \times \mathrm{SU}(2)$ symmetry. Their wave functions are associated with the following tensor bundles:

$$SU^{\pi}_{\rho}\overline{SU}^{\omega}_{\mu}M \otimes U^{\varepsilon}_{\eta}\bar{U}^{\sigma}_{\zeta}M \otimes D^{\alpha}_{\beta}\bar{D}^{\nu}_{\gamma}T^{m}_{n}M. \tag{4.1}$$

Smooth sections of the bundle (4.1) are called smooth spin-tensorial fields. Apart from the spin-tensorial type $(\alpha, \beta|\nu, \gamma|m, n)$, they are characterized by the electro-weak type $(\pi, \rho|\omega\mu|\varepsilon, \eta|\sigma\zeta)$. The following table is an electro-weak addendum to the table (1.2) containing the basic electro-weak fields:

Symbol	Name	Spin-tensorial and electro-weak types
$\mathbf{D}$	Hermitian metric tensor	$(0, 0\|0, 0\|0, 0)$ $(0, 0\|0, 0\|0, 1\|0, 1)$
$\mathbf{D}$	Hermitian metric tensor	$(0, 0\|0, 0\|0, 0)$ $(0, 1\|0, 1\|0, 0\|0, 0)$
$\mathbf{d}$	Skew-symmetric metric tensor	$(0, 0\|0, 0\|0, 0)$ $(0, 2\|0, 0\|0, 0\|0, 0)$

(4.2)

Quarks are described by the $\mathrm{U}(1) \times \mathrm{SU}(2) \times \mathrm{SU}(3)$ symmetry. Their wave functions are associated with the following tensor bundles:

$$SU^{\varkappa}_{\chi}\overline{SU}^{\varsigma}_{\chi}M \otimes SU^{\pi}_{\rho}\overline{SU}^{\omega}_{\mu}M \otimes U^{\varepsilon}_{\eta}\bar{U}^{\sigma}_{\zeta}M \otimes D^{\alpha}_{\beta}\bar{D}^{\nu}_{\gamma}T^{m}_{n}M. \tag{4.3}$$

Like in the case of the bundle (4.1), smooth sections of the bundle (4.3) are called smooth spin-tensorial fields. Apart from the spin-tensorial type $(\alpha, \beta|\nu, \gamma|m, n)$, they are characterized by the color type $(\varkappa, \lambda|\varsigma, \chi|\pi, \rho|\omega, \mu|\varepsilon, \eta|\sigma, \zeta)$. The following table is a color addendum to the tables (1.2) and (4.2):

Symbol	Name	Spin-tensorial and color types
$\mathbf{D}$	Hermitian metric tensor	$(0,0\|0,0\|0,0)$ $(0,0\|0,0\|0,0\|0,0\|0,1\|0,1)$
$\mathbf{D}$	Hermitian metric tensor	$(0,0\|0,0\|0,0)$ $(0,0\|0,0\|0,1\|0,1\|0,0\|0,0)$
$\mathbf{d}$	Skew-symmetric metric tensor	$(0,0\|0,0\|0,0)$ $(0,0\|0,0\|0,2\|0,0\|0,0\|0,0)$
$\mathbf{D}$	Hermitian metric tensor	$(0,0\|0,0\|0,0)$ $(0,1\|0,1\|0,0\|0,0\|0,0\|0,0)$
$\mathbf{d}$	Completely skew-symmetric tensor	$(0,0\|0,0\|0,0)$ $(0,3\|0,0\|0,0\|0,0\|0,0\|0,0)$

(4.4)

Note that U(1) and SU(2)-bundles in the case of leptons and in the case of quarks could be different (non-isomorphic) bundles. In this case the field $\mathbf{D}$, $\mathbf{D}$, $\mathbf{d}$ in the table (4.4) are different from $\mathbf{D}$, $\mathbf{D}$, $\mathbf{d}$ in the previous table (4.2). But even if it is so, it is convenient to denote these different fields by the same symbols.

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