COMMUTATION RELATIONSHIPS AND CURVATURE SPIN-TENSORS FOR EXTENDED SPINOR CONNECTIONS.

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Abstract. Extended spinor connections associated with composite spin-tensorial bundles are considered. Commutation relationships for covariant and multivariate differentiations and corresponding curvature spin-tensors are derived.

1. Introduction of a spinor bundle.

This paper is a continuation of the paper [1]. For this reason we keep all notations used in [1] and do not give any historical background referring the reader backward to the paper [1] and to the papers prior to it.

Let M be the *space-time* manifold, i. e. this is a 4-dimensional orientable manifold equipped with a pseudo-Euclidean metric $\mathbf{g}$ of the Minkowski-type signature $(+,-,-,-)$ and carrying a special smooth geometric structure which is called a *polarization*. Once some polarization is fixed, one can distinguish the *Future light cone* from the *Past light cone* at each point $p \in M$ (see [2] for more details). Moreover, we assume that M admits the spinor structure. This means that there is a two-dimensional smooth complex vector bundle SM over M equipped with a skew-symmetric spin-tensorial field $\mathbf{d}$. This spin-tensorial field $\mathbf{d}$ is called the *spin-metric tensor*, while SM is called the *spinor bundle*.

The spinor bundle SM differs from a general two-dimensional complex vector bundle over M by its close relation to the tangent bundle TM. The spin-metric tensor $\mathbf{d}$ and the metric tensor $\mathbf{g}$ are two basic structures establishing this relation. Any local trivialization of a two-dimensional vector bundle is given by two smooth sections of this bundle $\boldsymbol{\Psi}_1$ and $\boldsymbol{\Psi}_2$ which are $\mathbb{C}$-linearly independent at each point p of some open set $U \subset M$. These two spinor fields $\boldsymbol{\Psi}_1$ and $\boldsymbol{\Psi}_2$ form a moving frame $(U,\,\boldsymbol{\Psi}_1,\,\boldsymbol{\Psi}_2)$. A moving frame $(U,\,\boldsymbol{\Psi}_1,\,\boldsymbol{\Psi}_2)$ is called an orthonormal frame if

$$d_{ij} = d(\boldsymbol{\Psi}_i, \boldsymbol{\Psi}_j) = \begin{Vmatrix} 0 & 1 \\ -1 & 0 \end{Vmatrix}, \tag{1.1}$$

i. e. if the spin metric tensor $\mathbf{d}$ is given by the skew-symmetric matrix (1.1) in this frame. Similarly, a moving frame $(U,\,\boldsymbol{\Upsilon}_0,\,\boldsymbol{\Upsilon}_1,\,\boldsymbol{\Upsilon}_2,\,\boldsymbol{\Upsilon}_3)$ of the tangent bundle TM is given by four smooth vector fields $\boldsymbol{\Upsilon}_0$, $\boldsymbol{\Upsilon}_1$, $\boldsymbol{\Upsilon}_2$, $\boldsymbol{\Upsilon}_3$ which are $\mathbb{R}$-linearly independent at each point p of the open set $U \subset M$. This moving frame is called a

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positively polarized right orthonormal frame if the following conditions are fulfilled:

(1) the value of the first vector filed $\boldsymbol{\Upsilon}_0$ at each point $p \in U$ belongs to the interior of the Future light cone determined by the polarization of M;
(2) it is a right frame in the sense of the orientation of M;
(3) the metric tensor $\mathbf{g}$ is given by the standard Minkowski matrix in this frame:

$$g_{ij} = g(\boldsymbol{\Upsilon}_i, \boldsymbol{\Upsilon}_j) = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{Vmatrix}. \tag{1.2}$$

If we have two orthonormal moving frames $(U,\, \boldsymbol{\Psi}_1,\, \boldsymbol{\Psi}_2)$ and $(\tilde{U},\, \tilde{\boldsymbol{\Psi}}_1,\, \tilde{\boldsymbol{\Psi}}_2)$ of the spinor bundle SM with overlapping domains $U \cap \tilde{U} \neq \varnothing$, then at each point $p \in U \cap \tilde{U}$ we have the following transition formulas:

$$\tilde{\boldsymbol{\Psi}}_i = \sum_{j=1}^{2} \mathfrak{S}^j_i\, \boldsymbol{\Psi}_j, \qquad\qquad \boldsymbol{\Psi}_i = \sum_{j=1}^{2} \mathfrak{T}^j_i\, \tilde{\boldsymbol{\Psi}}_j, \tag{1.3}$$

The transition matrices $\mathfrak{S}$ and $\mathfrak{T}$ in (1.3) are inverse to each other: $\mathfrak{T} = \mathfrak{S}^{-1}$. From (1.1) for these matrices one easily derives

$$\mathfrak{S}(p) \in \mathrm{SL}(2, \mathbb{C}), \qquad\qquad \mathfrak{T}(p) \in \mathrm{SL}(2, \mathbb{C}). \tag{1.4}$$

In a similar way, if we have two positively polarized right orthonormal frames $(U,\, \boldsymbol{\Upsilon}_0,\, \boldsymbol{\Upsilon}_1,\, \boldsymbol{\Upsilon}_2,\, \boldsymbol{\Upsilon}_3)$ and $(\tilde{U},\, \tilde{\boldsymbol{\Upsilon}}_0,\, \tilde{\boldsymbol{\Upsilon}}_1,\, \tilde{\boldsymbol{\Upsilon}}_2,\, \tilde{\boldsymbol{\Upsilon}}_3)$ of the tangent bundle TM with overlapping domains $U \cap \tilde{U} \neq \varnothing$, then we have

$$\tilde{\boldsymbol{\Upsilon}}_i = \sum_{j=0}^{3} S^j_i\, \boldsymbol{\Upsilon}_j, \qquad\qquad \boldsymbol{\Upsilon}_i = \sum_{j=0}^{3} T^j_i\, \tilde{\boldsymbol{\Upsilon}}_j. \tag{1.5}$$

The matrices S and T in the formulas (1.5) are inverse to each other: $T = S^{-1}$. From (1.2) and from the above conditions (1)–(3) for each point $p \in U \cap \tilde{U}$ we get

$$S(p) \in \mathrm{SO}^+(1, 3, \mathbb{R}), \qquad\qquad T(p) \in \mathrm{SO}^+(1, 3, \mathbb{R}).$$

Note that the special linear group $\mathrm{SL}(2, \mathbb{C})$ in (1.4) and the special orthochronous Lorentzian group $\mathrm{SO}^+(1, 3, \mathbb{R})$ are related by the canonical homomorphism

$$\varphi\colon\ \mathrm{SL}(2, \mathbb{C}) \to \mathrm{SO}^+(1, 3, \mathbb{R}). \tag{1.6}$$

The canonical homomorphism φ in (1.6) is a surjective mapping. Its kernel is a discrete group. It is composed by the following two matrices:

$$\boldsymbol{\sigma}_0 = \begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix}, \qquad\qquad -\boldsymbol{\sigma}_0 = \begin{Vmatrix} -1 & 0 \\ 0 & -1 \end{Vmatrix}. \tag{1.7}$$

Since $\operatorname{Ker}\varphi$ is a discrete set composed by two matrices (1.7), topologically φ is a double sheeted not ramified covering of real analytic manifolds. It can be given by explicit formulas in terms of the matrix components. If

$$S=\varphi(\mathfrak{S}),$$

then for the components of the matrix S we have the following expressions:

$$\begin{aligned}
S^0_0&=\frac{\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^1_1+\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^1_2+\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^2_1+\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^2_2}{2},\\
S^0_1&=\frac{\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^1_2+\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^1_1+\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^2_2+\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^2_1}{2},\\
S^0_2&=\frac{\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^1_1-\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^1_2+\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^2_1-\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^2_2}{2\,i},\\
S^0_3&=\frac{\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^1_1-\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^1_2+\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^2_1-\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^2_2}{2},
\end{aligned}\tag{1.8}$$

$$\begin{aligned}
S^1_0&=\frac{\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^1_1+\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^2_1+\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^1_2+\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^2_2}{2},\\
S^1_1&=\frac{\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^1_2+\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^2_1+\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^1_1+\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^2_2}{2},\\
S^1_2&=\frac{\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^2_1-\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^1_2+\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^1_1-\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^2_2}{2\,i},\\
S^1_3&=\frac{\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^1_1+\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^2_1-\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^1_2-\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^2_2}{2},
\end{aligned}\tag{1.9}$$

$$\begin{aligned}
S^2_0&=\frac{\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^2_1-\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^1_1+\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^2_2-\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^1_2}{2\,i},\\
S^2_1&=\frac{\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^2_1-\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^1_2+\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^2_2-\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^1_1}{2\,i},\\
S^2_2&=\frac{\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^1_1+\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^2_2-\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^1_2-\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^2_1}{2},\\
S^2_3&=\frac{\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^2_1-\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^1_1+\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^1_2-\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^2_2}{2\,i},
\end{aligned}\tag{1.10}$$

$$\begin{aligned}
S^3_0&=\frac{\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^1_1+\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^1_2-\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^2_1-\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^2_2}{2},\\
S^3_1&=\frac{\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^1_2+\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^1_1-\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^2_2-\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^2_1}{2},\\
S^3_2&=\frac{\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^1_1-\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^1_2+\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^2_2-\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^2_1}{2\,i},\\
S^3_3&=\frac{\overline{\mathfrak{S}^1_1}\,\mathfrak{S}^1_1+\overline{\mathfrak{S}^2_2}\,\mathfrak{S}^2_2-\overline{\mathfrak{S}^2_1}\,\mathfrak{S}^2_1-\overline{\mathfrak{S}^1_2}\,\mathfrak{S}^1_2}{2}.
\end{aligned}\tag{1.11}$$

The first formula (1.8) was presented in [1]. The whole set of the above formulas (1.8), (1.9), (1.10), and (1.11) can be found in [3].

Definition 1.1. Let SM be a two-dimensional complex vector bundle over the space-time manifold M equipped with a nonzero spin-metric $\mathbf{d}$. It is called the *spinor bundle* if each orthonormal frame $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2)$ of SM is associated with some positively polarized right orthonormal frame $(U,\,\mathbf{\Upsilon}_0,\,\mathbf{\Upsilon}_1,\,\mathbf{\Upsilon}_2,\,\mathbf{\Upsilon}_3)$ of the tangent bundle TM such that for any two orthonormal frames $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2)$ and $(\tilde U,\,\tilde{\mathbf{\Psi}}_1,\,\tilde{\mathbf{\Psi}}_2)$ with overlapping domains $U\cap\tilde U\neq\varnothing$ the associated tangent frames $(U,\,\mathbf{\Upsilon}_0,\,\mathbf{\Upsilon}_1,\,\mathbf{\Upsilon}_2,\,\mathbf{\Upsilon}_3)$ and $(\tilde U,\,\tilde{\mathbf{\Upsilon}}_0,\,\tilde{\mathbf{\Upsilon}}_1,\,\tilde{\mathbf{\Upsilon}}_2,\,\tilde{\mathbf{\Upsilon}}_3)$ are related to each other by means of the formulas (1.5), where the transition matrices S and T are obtained from the transition matrices $\mathfrak{S}$ and $\mathfrak{T}$ in (1.3) by applying the group homomorphism (1.6), i. e. $S=\varphi(\mathfrak{S})$ and $T=\varphi(\mathfrak{T})$.

Let $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2)$ be an orthonormal frame of the spinor bundle SM and assume that the domain U is sufficiently small to be equipped with some local coordinates x^0, x^1, x^2, x^3. Then U is a local chart and we have the holonomic frame in TM determined by the local coordinates of this chart:

$$\mathbf{E}_0=\frac{\partial}{\partial x^0},\quad \mathbf{E}_1=\frac{\partial}{\partial x^1},\quad \mathbf{E}_2=\frac{\partial}{\partial x^2},\quad \mathbf{E}_3=\frac{\partial}{\partial x^3}. \tag{1.12}$$

Passing from $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2)$ to the associated frame $(U,\,\mathbf{\Upsilon}_0,\,\mathbf{\Upsilon}_1,\,\mathbf{\Upsilon}_2,\,\mathbf{\Upsilon}_3)$, in general case we find that it doesn't coincide with the coordinate frame $(U,\,\mathbf{E}_0,\,\mathbf{E}_1,\,\mathbf{E}_2,\,\mathbf{E}_3)$ since in general case $(U,\,\mathbf{\Upsilon}_0,\,\mathbf{\Upsilon}_1,\,\mathbf{\Upsilon}_2,\,\mathbf{\Upsilon}_3)$ is a non-holonomic frame. Let's consider the following expansion of the non-holonomic frame vectors:

$$\mathbf{\Upsilon}_i=\sum_{j=0}^{3}\Upsilon^j_i\,\mathbf{E}_j. \tag{1.13}$$

In the case of a holonomic frame (1.12) all mutual commutators of the vector fields $\mathbf{E}_0$, $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$ are equal to zero. In the case of a non-holonomic frame it is not so:

$$[\mathbf{\Upsilon}_i,\,\mathbf{\Upsilon}_j]=\sum_{k=0}^{3}c^k_{ij}\,\mathbf{\Upsilon}_k. \tag{1.14}$$

From (1.13) and (1.14) one easily derives

$$\sum_{s=0}^{3}\Upsilon^s_i\,\frac{\partial\Upsilon^m_j}{\partial x^s}-\sum_{s=0}^{3}\Upsilon^s_j\,\frac{\partial\Upsilon^m_i}{\partial x^s}=\sum_{k=0}^{3}c^k_{ij}\,\Upsilon^m_k. \tag{1.15}$$

The coefficients c^k_{ij} can be calculated using either (1.14) or (1.15). They form a frame specific set of functions.

2. Tensors and spin-tensors.

Let M be the space-time and let p be a point of M. Then $T_p(M)$ is a tangent space of M at the point p. Similarly, $T^*_p(M)$ is a cotangent space at the same point, it is dual to the space $T_p(M)$. Both $T_p(M)$ and $T^*_p(M)$ are real linear vector spaces. Following the recipe of [1], we introduce their complexifications:

$$\mathbb{C}T_p(M)=\mathbb{C}\otimes T_p(M),\qquad\qquad \mathbb{C}T^*_p(M)=\mathbb{C}\otimes T^*_p(M). \tag{2.1}$$

Complexified tensor spaces then are introduced as multiple tensor products of several copies of the spaces $\mathbb{C}T_p(M)$ and $\mathbb{C}T^*_p(M)$ introduced in (2.1):

$$\mathbb{C}T^m_n(p,M)=\overbrace{\mathbb{C}T_p(M)\otimes\ldots\otimes\mathbb{C}T_p(M)}^{m\text{ times}}\otimes\underbrace{\mathbb{C}T^*_p(M)\otimes\ldots\otimes\mathbb{C}T^*_p(M)}_{n\text{ times}}. \tag{2.2}$$

The spinor bundle SM is a complex vector bundle over M from the very beginning. Let's denote by $S_p(M)$ its fiber over the point $p\in M$. Let $S^*_p(M)$ be its dual space. Moreover, we consider the hermitian conjugate space $S^\dagger_p(M)$ for $S_p(M)$ and its dual space $S^{\dagger *}_p(M)=S^{*\dagger}_p(M)$. Then we can define the following tensor products:

$$S^\alpha_\beta(p,M)=\overbrace{S_p(M)\otimes\ldots\otimes S_p(M)}^{\alpha\text{ times}}\otimes\underbrace{S^*_p(M)\otimes\ldots\otimes S^*_p(M)}_{\beta\text{ times}}, \tag{2.3}$$

$$\bar{S}^\nu_\gamma(p,M)=\overbrace{S^{\dagger *}_p(M)\otimes\ldots\otimes S^{\dagger *}_p(M)}^{\nu\text{ times}}\otimes\underbrace{S^\dagger_p(M)\otimes\ldots\otimes S^\dagger_p(M)}_{\gamma\text{ times}}. \tag{2.4}$$

Combining (2.2), (2.3), and (2.4), we define one more tensor product

$$S^\alpha_\beta\bar{S}^\nu_\gamma T^r_s(p,M)=S^\alpha_\beta(p,M)\otimes\bar{S}^\nu_\gamma(p,M)\otimes\mathbb{C}T^m_n(p,M). \tag{2.5}$$

Elements of the space (2.5) are called *spin-tensors* of the type $(\alpha,\beta|\nu,\gamma|m,n)$ at the point $p\in M$. Elements of other three spaces (2.2), (2.3), and (2.4) are also called spin-tensors, though they are special cases of a general spin-tensorial object. The spin-tensorial space (2.5) admits the canonical semilinear isomorphism τ:

$$\tau\colon\ S^\alpha_\beta\bar{S}^\nu_\gamma T^m_n(p,M)\to S^\nu_\gamma\bar{S}^\alpha_\beta T^m_n(p,M) \tag{2.6}$$

(see the definition of τ and more details concerning it in [1]). The spin-tensorial spaces (2.5) with p running over M are glued into a bundle. It is called the *spin-tensorial bundle* of the type $(\alpha,\beta|\nu,\gamma|m,n)$ and is denoted by $S^\alpha_\beta\bar{S}^\nu_\gamma T^m_nM$. Then the isomorphisms (2.6) with the point p running over M are glued into a semilinear isomorphism of spin-tensorial bundles:

$$\tau\colon\ S^\alpha_\beta\bar{S}^\nu_\gamma T^m_nM\to S^\nu_\gamma\bar{S}^\alpha_\beta T^m_nM.$$

A traditional spin-tensorial field of the type $(\alpha,\beta|\nu,\gamma|m,n)$ by definition is a local or global smooth section of the spin-tensorial bundle $S^\alpha_\beta\bar{S}^\nu_\gamma T^m_nM$. Non-traditional (extended) spin-tensorial fields were introduced in [1] along with non-traditional (extended) connections. We shall give their definitions a little bit later.

3. Coordinate representation of spin-tensorial fields.

In order to represent a tensorial field in a coordinate form it is sufficient to have a local chart in M. In the case of spin-tensorial fields, in addition to a local chart, we need to have two frames: one in SM and the other in TM.

Definition 3.1. Let U be a local chart of the space-time manifold M. We say that U is an *equipped local chart* if there is an orthonormal spinor frame $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2)$ with the domain U and, hence, according to the definition 1.1, there is a positively polarized right orthonormal tangent frame $(U,\,\mathbf{\Upsilon}_0,\,\mathbf{\Upsilon}_1,\,\mathbf{\Upsilon}_2,\,\mathbf{\Upsilon}_3)$ canonically associated with the frame $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2)$.

Equipped local charts cover the whole space-time manifold, i. e. they form an atlas. Therefore, they describe completely the structure of the the space-time manifold M and its bundles SM and TM.

Let U be an equipped local chart with the local coordinates $x^0,\,x^1,\,x^2,\,x^3$. Assume that $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2)$ and $(U,\,\mathbf{\Upsilon}_0,\,\mathbf{\Upsilon}_1,\,\mathbf{\Upsilon}_2,\,\mathbf{\Upsilon}_3)$ are two frames associated with U and thus being its equipment. Denote by $(U,\,\boldsymbol{\vartheta}^1,\,\boldsymbol{\vartheta}^2)$ the dual cospinor frame for $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2)$ and denote by $(U,\,\boldsymbol{\eta}^0,\,\boldsymbol{\eta}^1,\,\boldsymbol{\eta}^2,\,\boldsymbol{\eta}^3)$ the dual covectorial frame for $(U,\,\mathbf{\Upsilon}_0,\,\mathbf{\Upsilon}_1,\,\mathbf{\Upsilon}_2,\,\mathbf{\Upsilon}_3)$. Moreover, we denote

$$\overline{\mathbf{\Psi}}_i=\tau(\mathbf{\Psi}_i), \qquad\qquad \overline{\boldsymbol{\vartheta}}^{\,i}=\tau(\boldsymbol{\vartheta}^{\,i}). \tag{3.1}$$

The barred spinor fields (3.1) form two frames $(U,\,\overline{\mathbf{\Psi}}_1,\,\overline{\mathbf{\Psi}}_2)$ and $(U,\,\overline{\boldsymbol{\vartheta}}^{\,1},\,\overline{\boldsymbol{\vartheta}}^{\,2})$ dual to each other. Using all the above frames, we define the following fields:

$$\mathbf{\Upsilon}^{k_1\dots\,k_n}_{h_1\dots\,h_m}=\mathbf{\Upsilon}_{h_1}\otimes\ldots\otimes\mathbf{\Upsilon}_{h_m}\otimes\boldsymbol{\eta}^{k_1}\otimes\ldots\otimes\boldsymbol{\eta}^{k_n}, \tag{3.2}$$

$$\mathbf{\Psi}^{j_1\dots\,j_\beta}_{i_1\dots\,i_\alpha}=\mathbf{\Psi}_{i_1}\otimes\ldots\otimes\mathbf{\Psi}_{i_\alpha}\otimes\boldsymbol{\vartheta}^{j_1}\otimes\ldots\otimes\boldsymbol{\vartheta}^{j_\beta}, \tag{3.3}$$

$$\overline{\mathbf{\Psi}}^{\bar{j}_1\dots\,\bar{j}_\gamma}_{\bar{i}_1\dots\,\bar{i}_\nu}=\overline{\mathbf{\Psi}}_{\bar{i}_1}\otimes\ldots\otimes\overline{\mathbf{\Psi}}_{\bar{i}_\nu}\otimes\overline{\boldsymbol{\vartheta}}^{j_1}\otimes\ldots\otimes\overline{\boldsymbol{\vartheta}}^{j_\gamma}. \tag{3.4}$$

Then, using (3.2), (3.3), and (3.4), we introduce the following tensor product:

$$\mathbf{\Psi}^{j_1\dots\,j_\beta\,\bar{j}_1\dots\,\bar{j}_\gamma\,k_1\dots\,k_n}_{i_1\dots\,i_\alpha\,\bar{i}_1\dots\,\bar{i}_\nu\,h_1\dots\,h_m}=\mathbf{\Psi}^{j_1\dots\,j_\beta}_{i_1\dots\,i_\alpha}\otimes\overline{\mathbf{\Psi}}^{\bar{j}_1\dots\,\bar{j}_\gamma}_{\bar{i}_1\dots\,\bar{i}_\nu}\otimes\mathbf{\Upsilon}^{k_1\dots\,k_n}_{h_1\dots\,h_m}. \tag{3.5}$$

It is easy to see that the formula (3.5) defines a series of local spin-tensorial fields of the type $(\alpha,\beta|\nu,\gamma|m,n)$ with the domain U. Assume that $\mathbf{X}$ is an arbitrary spin-tensorial field of the same type $(\alpha,\beta|\nu,\gamma|m,n)$. For this spin-tensorial field within the domain U we can write the following expansion:

$$\mathbf{X}=\sum^2_{\substack{i_1,\,\dots\,,\,i_\alpha\\ j_1,\,\dots\,,\,j_\beta}}\!\!\dots\!\sum^2\ \sum^2_{\substack{\bar{i}_1,\,\dots\,,\,\bar{i}_\nu\\ \bar{j}_1,\,\dots\,,\,\bar{j}_\gamma}}\!\!\dots\!\sum^2\ \sum^3_{\substack{h_1,\,\dots\,,\,h_m\\ k_1,\,\dots\,,\,k_n}}\!\!\dots\!\sum^3 X^{i_1\dots\,i_\alpha\,\bar{i}_1\dots\,\bar{i}_\nu\,h_1\dots\,h_m}_{j_1\dots\,j_\beta\,\bar{j}_1\dots\,\bar{j}_\gamma\,k_1\dots\,k_n}\ \mathbf{\Psi}^{j_1\dots\,j_\beta\,\bar{j}_1\dots\,\bar{j}_\gamma\,k_1\dots\,k_n}_{i_1\dots\,i_\alpha\,\bar{i}_1\dots\,\bar{i}_\nu\,h_1\dots\,h_m}. \tag{3.6}$$

The coefficients $X^{i_1\dots\,i_\alpha\,\bar{i}_1\dots\,\bar{i}_\nu\,h_1\dots\,h_m}_{j_1\dots\,j_\beta\,\bar{j}_1\dots\,\bar{j}_\gamma\,k_1\dots\,k_n}$ in the expansion (3.6) are functions of the local coordinates $x^0,\,x^1,\,x^2,\,x^3$ of a point $p\in U$:

$$X^{i_1\dots\,i_\alpha\,\bar{i}_1\dots\,\bar{i}_\nu\,h_1\dots\,h_m}_{j_1\dots\,j_\beta\,\bar{j}_1\dots\,\bar{j}_\gamma\,k_1\dots\,k_n}=X^{i_1\dots\,i_\alpha\,\bar{i}_1\dots\,\bar{i}_\nu\,h_1\dots\,h_m}_{j_1\dots\,j_\beta\,\bar{j}_1\dots\,\bar{j}_\gamma\,k_1\dots\,k_n}(x^0,x^1,x^2,x^3). \tag{3.7}$$

These functions (3.7) are called the *components* of a spin-tensorial field X in an equipped local chart U, while the expansion (3.6) itself is the *coordinate representation* of the field $\mathbf{X}$.

4. Composite spin-tensorial bundles and extended spin-tensorial fields.

Let $\mathbf{S}[1],\,\ldots\,,\mathbf{S}[J]$ be some spin-tensorial fields of various types, e. g. we can denote by $(\alpha_P,\beta_P|\nu_P,\gamma_P|m_P,n_P)$ the type of the P-th field $\mathbf{S}[P]$ in the series $\mathbf{S}[1],\,\ldots\,,\mathbf{S}[J]$. In some cases one need to treat $\mathbf{S}[1],\,\ldots\,,\mathbf{S}[J]$ not as actual fields, but as independent variables. For example, if we consider a physical field theory with the fields $\mathbf{S}[1],\,\ldots\,,\mathbf{S}[J]$, then the Lagrange function

$$\mathcal{L}=\mathcal{L}(p,\mathbf{S}[1],\ldots,\mathbf{S}[J]) \tag{4.1}$$

of this field theory is a function of several spin-tensorial arguments $\mathbf{S}[1],\,\ldots\,,\mathbf{S}[J]$ and of one point argument $p\in M$. Composite spin-tensorial bundles were introduced in [1] for to formalize the argument set of the function (4.1). In the present case the composite tensor bundle N is defined as the following direct sum:

$$N=S^{\alpha_1}_{\beta_1}\bar S^{\nu_1}_{\gamma_1}T^{m_1}_{n_1}M\oplus\ldots\oplus S^{\alpha_J}_{\beta_J}\bar S^{\nu_J}_{\gamma_J}T^{m_J}_{n_J}M. \tag{4.2}$$

By definition a point q of the composite spin-tensorial bundle (4.2) is a list

$$q=(p,\,\mathbf{S}[1],\,\ldots\,,\,\mathbf{S}[J]), \tag{4.3}$$

where p is a point of the space-time M, while $\mathbf{S}[1],\,\ldots\,,\mathbf{S}[J]$ are spin-tensors of the types $(\alpha_1,\beta_1|\nu_1,\gamma_1|m_1,n_1),\,\ldots\,,(\alpha_J,\beta_J|\nu_J,\gamma_J|m_J,n_J)$ at the point p.

Definition 4.1. Let N be a composite spin-tensorial bundle over the space-time manifold M in the sense of the formula (4.2). An extended spin-tensorial field $\mathbf{X}$ of the type $(\varepsilon,\eta|\sigma,\zeta|e,f)$ is a spin-tensor-valued function in N such that it takes each point $q\in N$ to some spin-tensor $\mathbf{X}(q)\in S^{\varepsilon}_{\eta}\bar S^{\sigma}_{\zeta}T^{e}_{f}(p,M)$, where $p=\pi(q)$ is the projection of the point q.

Let U be an equipped local chart of the space-time manifold M (see the definition 3.1 above). Then a point $p\in U$ is given by its coordinates

$$x^0,\,x^1,\,x^2,\,x^3, \tag{4.4}$$

while spin-tensors $\mathbf{S}[1],\,\ldots\,,\mathbf{S}[J]$ at the point p are given by their components referred to the frames $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2)$ and $(U,\,\mathbf{\Upsilon}_0,\,\mathbf{\Upsilon}_1,\,\mathbf{\Upsilon}_2,\,\mathbf{\Upsilon}_3)$:

$$\begin{gathered}S^{1\ldots1\,1\ldots1\,0\ldots0}_{1\ldots1\,1\ldots1\,0\ldots0}[1],\,\ldots\,,\,S^{2\ldots2\,2\ldots2\,3\ldots3}_{2\ldots2\,2\ldots2\,3\ldots3}[1],\,\ldots\\ \ldots\,,\,S^{1\ldots1\,1\ldots1\,0\ldots0}_{1\ldots1\,1\ldots1\,0\ldots0}[J],\,\ldots\,,\,S^{2\ldots2\,2\ldots2\,3\ldots3}_{2\ldots2\,2\ldots2\,3\ldots3}[J].\end{gathered} \tag{4.5}$$

The quantities (4.4) and (4.5) form a complete set of variables that can be used as local coordinates for a point q in (4.3). For an extended spin-tensorial field $\mathbf{X}$ of the type $(\varepsilon,\eta|\sigma,\zeta|e,f)$ one can write the expansion

$$\mathbf{X}=\sum^{2}_{\substack{i_1,\,\ldots\,,\,i_\varepsilon\\ j_1,\,\ldots\,,\,j_\eta}}\!\!\ldots\!\sum^{2}\,\sum^{2}_{\substack{\bar i_1,\,\ldots\,,\,\bar i_\sigma\\ \bar j_1,\,\ldots\,,\,\bar j_\zeta}}\!\!\ldots\!\sum^{2}\,\sum^{3}_{\substack{h_1,\,\ldots\,,\,h_e\\ k_1,\,\ldots\,,\,k_f}}\!\!\ldots\!\sum^{3}X^{i_1\ldots\,i_\varepsilon\,\bar i_1\ldots\,\bar i_\sigma\,h_1\ldots\,h_e}_{j_1\ldots\,j_\eta\,\bar j_1\ldots\,\bar j_\zeta\,k_1\ldots\,k_f}\;\mathbf{\Psi}^{j_1\ldots\,j_\eta\,\bar j_1\ldots\,\bar j_\zeta\,k_1\ldots\,k_f}_{i_1\ldots\,i_\varepsilon\,\bar i_1\ldots\,\bar i_\sigma\,h_1\ldots\,h_e} \tag{4.6}$$

similar to (3.6). However, unlike to (3.7), the coefficients $X^{i_1\ldots\,i_\varepsilon\,\bar{i}_1\ldots\,\bar{i}_\sigma\,h_1\ldots\,h_e}_{j_1\ldots\,j_\eta\,\bar{j}_1\ldots\,\bar{j}_\varsigma\,k_1\ldots\,k_f}$ in the expansion (4.6) are functions of the whole set of variables (4.4) and (4.5). Thus, taking an equipped local chart U of M, we get a coordinate representation for points of the composite spin-tensorial bundle (4.2) and for extended spin-tensorial fields associated with it.

Under a change of equipped local charts the coordinates (4.4) are transformed traditionally by means of the transition functions

$$\begin{cases}\tilde{x}^0=\tilde{x}^1(x^0,\,\ldots\,,x^3),\\ \ldots\ldots\ldots\ldots\ldots\\ \tilde{x}^3=\tilde{x}^n(x^0,\,\ldots\,,x^3).\end{cases}\qquad\qquad \begin{cases}x^0=\tilde{x}^1(\tilde{x}^0,\,\ldots\,,\tilde{x}^3),\\ \ldots\ldots\ldots\ldots\ldots\\ x^3=\tilde{x}^n(\tilde{x}^0,\,\ldots\,,\tilde{x}^3),\end{cases}$$

while the coordinates (4.5) are transformed as the components of spin-tensors

$$\begin{cases}\tilde{S}^{i_1\ldots\,i_\alpha\bar{i}_1\ldots\,\bar{i}_\nu h_1\ldots\,h_m}_{j_1\ldots\,j_\beta\bar{j}_1\ldots\,\bar{j}_\gamma k_1\ldots\,k_n}[P]=\sum^{2}_{\substack{a_1,\,\ldots\,,\,a_\alpha\\ b_1,\,\ldots\,,\,b_\beta}}\!\!\ldots\!\sum^{2}\sum^{2}_{\substack{\bar{a}_1,\,\ldots\,,\,\bar{a}_\nu\\ \bar{b}_1,\,\ldots\,,\,\bar{b}_\gamma}}\!\!\ldots\!\sum^{2}\sum^{3}_{\substack{c_1,\,\ldots\,,\,c_m\\ d_1,\,\ldots\,,\,d_n}}\!\!\ldots\!\sum^{3}\mathfrak{T}^{i_1}_{a_1}\ldots\,\mathfrak{T}^{i_\alpha}_{a_\alpha}\times\\ \qquad\times\ \mathfrak{S}^{b_1}_{j_1}\ldots\,\mathfrak{S}^{b_\beta}_{j_\beta}\ \overline{\mathfrak{T}^{\bar{i}_1}_{\bar{a}_1}}\ldots\,\overline{\mathfrak{T}^{\bar{i}_\nu}_{\bar{a}_\nu}}\ \ \overline{\mathfrak{S}^{\bar{b}_1}_{\bar{j}_1}}\ldots\,\overline{\mathfrak{S}^{\bar{b}_\gamma}_{\bar{j}_\gamma}}\ T^{h_1}_{c_1}\ldots\,T^{h_m}_{c_m}\times\\ \qquad\times\ S^{d_1}_{k_1}\ldots\,S^{d_n}_{k_n}\ S^{a_1\ldots\,a_\alpha\,\bar{a}_1\ldots\,\bar{a}_\nu\,c_1\ldots\,c_m}_{b_1\ldots\,b_\beta\,\bar{b}_1\ldots\,\bar{b}_\gamma\,d_1\ldots\,d_n}[P],\end{cases}\tag{4.7}$$

$$\begin{cases}S^{i_1\ldots\,i_\alpha\bar{i}_1\ldots\,\bar{i}_\nu h_1\ldots\,h_m}_{j_1\ldots\,j_\beta\bar{j}_1\ldots\,\bar{j}_\gamma k_1\ldots\,k_n}[P]=\sum^{2}_{\substack{a_1,\,\ldots\,,\,a_\alpha\\ b_1,\,\ldots\,,\,b_\beta}}\!\!\ldots\!\sum^{2}\sum^{2}_{\substack{\bar{a}_1,\,\ldots\,,\,\bar{a}_\nu\\ \bar{b}_1,\,\ldots\,,\,\bar{b}_\gamma}}\!\!\ldots\!\sum^{2}\sum^{3}_{\substack{c_1,\,\ldots\,,\,c_m\\ d_1,\,\ldots\,,\,d_n}}\!\!\ldots\!\sum^{3}\mathfrak{S}^{i_1}_{a_1}\ldots\,\mathfrak{S}^{i_\alpha}_{a_\alpha}\times\\ \qquad\times\ \mathfrak{T}^{b_1}_{j_1}\ldots\,\mathfrak{T}^{b_\beta}_{j_\beta}\ \overline{\mathfrak{S}^{\bar{i}_1}_{\bar{a}_1}}\ldots\,\overline{\mathfrak{S}^{\bar{i}_\nu}_{\bar{a}_\nu}}\ \ \overline{\mathfrak{T}^{\bar{b}_1}_{\bar{j}_1}}\ldots\,\overline{\mathfrak{T}^{\bar{b}_\gamma}_{\bar{j}_\gamma}}\ S^{h_1}_{c_1}\ldots\,S^{h_m}_{c_m}\times\\ \qquad\times\ T^{d_1}_{k_1}\ldots\,T^{d_n}_{k_n}\ \tilde{S}^{a_1\ldots\,a_\alpha\,\bar{a}_1\ldots\,\bar{a}_\nu\,c_1\ldots\,c_m}_{b_1\ldots\,b_\beta\,\bar{b}_1\ldots\,\bar{b}_\gamma\,d_1\ldots\,d_n}[P],\end{cases}\tag{4.8}$$

where $\alpha=\alpha_P$, $\beta=\beta_P$, $\nu=\nu_P$, $\gamma=\gamma_P$, $m=m_P$, $n=n_P$, and the integer number P runs from 1 to J. The components of transition matrices $\mathfrak{S}$, $\mathfrak{T}$, $S=\varphi(\mathfrak{S})$, and $T=\varphi(\mathfrak{T})$ in (4.7) and (4.8) are taken from the frame relationships (1.3) and (1.5).

5. Extended spinor connections.

Extended spinor connections were introduced in [1] in order to describe the structure of differentiations of extended spin-tensorial fields. In order to define them here we consider a pair of equipped local charts U and $\tilde{U}$ with non-empty intersection. Then we introduce the following θ-parameters defined within $U\cap\tilde{U}$:

$$\tilde{\theta}^k_{ij}=\sum^3_{a=0}T^k_a\,L_{\tilde{\Upsilon}_i}(S^a_j)=\sum^3_{a=0}\sum^3_{v=0}T^k_a\,\tilde{\Upsilon}^v_i\,\frac{\partial S^a_j}{\partial\tilde{x}^v}=-\sum^3_{a=0}L_{\tilde{\Upsilon}_i}(T^k_a)\,S^a_j,\tag{5.1}$$

$$\tilde{\vartheta}^k_{ij}=\sum^2_{a=1}\mathfrak{T}^k_a\,L_{\tilde{\Upsilon}_i}(\mathfrak{S}^a_j)=\sum^2_{a=1}\sum^3_{v=0}\mathfrak{T}^k_a\,\tilde{\Upsilon}^v_i\,\frac{\partial\mathfrak{S}^a_j}{\partial\tilde{x}^v}=-\sum^2_{a=1}L_{\tilde{\Upsilon}_i}(\mathfrak{T}^k_a)\,\mathfrak{S}^a_j.\tag{5.2}$$

The θ-parameters without tilde are introduced in a similar way:

$$\theta^k_{ij}=\sum_{a=0}^{3}S^k_a\,L_{\boldsymbol{\Upsilon}_i}(T^a_j)=\sum_{a=0}^{3}\sum_{v=0}^{3}S^k_a\,\Upsilon^v_i\,\frac{\partial T^a_j}{\partial x^v}=-\sum_{a=0}^{3}L_{\boldsymbol{\Upsilon}_i}(S^k_a)\,T^a_j,\tag{5.3}$$

$$\vartheta^k_{ij}=\sum_{a=1}^{2}\mathfrak{S}^k_a\,L_{\boldsymbol{\Upsilon}_i}(\mathfrak{T}^a_j)=\sum_{a=1}^{2}\sum_{v=0}^{3}\mathfrak{S}^k_a\,\Upsilon^v_i\,\frac{\partial \mathfrak{T}^a_j}{\partial \tilde{x}^v}=-\sum_{a=1}^{2}L_{\boldsymbol{\Upsilon}_i}(\mathfrak{S}^k_a)\,\mathfrak{T}^a_j.\tag{5.4}$$

Note that $L_{\tilde{\boldsymbol{\Upsilon}}_i}$ and $L_{\boldsymbol{\Upsilon}_i}$ in (5.1), (5.2), (5.3), and (5.4) are Lie derivatives applied to scalar functions T^k_a, $\mathfrak{T}^k_a$, S^k_a, and $\mathfrak{S}^k_a$ respectively. When applied to an arbitrary scalar function f in $U\cap\tilde{U}$, the Lie derivative $L_{\boldsymbol{\Upsilon}_i}$ acts as follows:

$$L_{\boldsymbol{\Upsilon}_i}(f)=\sum_{j=0}^{3}\Upsilon^j_i\,\frac{\partial f}{\partial x^j}.\tag{5.5}$$

The coefficients Υ^j_i in (5.5) coincide with those in the expansion (1.13).

Definition 5.1. Let N be the composite spin-tensorial bundle (4.2) over the space-time manifold M. An extended spinor connection is a geometric object such that in each equipped local chart U of M it is represented by its components $\mathrm{A}^k_{j\,i}$, $\bar{\mathrm{A}}^k_{j\,i}$, $\Gamma^k_{j\,i}$ and such that its components are smooth functions of the variables (4.4) and (4.5) transforming according to the formulas

$$\mathrm{A}^k_{j\,i}=\sum_{b=1}^{2}\sum_{a=1}^{2}\sum_{c=0}^{3}\mathfrak{S}^k_a\,\mathfrak{T}^b_i\,T^c_j\;\tilde{\mathrm{A}}^a_{c\,b}+\vartheta^k_{j\,i},\tag{5.6}$$

$$\bar{\mathrm{A}}^k_{j\,i}=\sum_{b=1}^{2}\sum_{a=1}^{2}\sum_{c=0}^{3}\overline{\mathfrak{S}^k_a}\;\overline{\mathfrak{T}^b_i}\,T^c_j\;\tilde{\bar{\mathrm{A}}}^a_b+\overline{\vartheta^k_{j\,i}},\tag{5.7}$$

$$\Gamma^k_{j\,i}=\sum_{b=0}^{3}\sum_{a=0}^{3}\sum_{c=0}^{3}S^k_a\,T^b_i\,T^c_j\;\tilde{\Gamma}^a_{c\,b}+\theta^k_{j\,i}\tag{5.8}$$

under a change of a local chart. The θ-parameters in the transformation formulas (5.6), (5.7), and (5.8) are taken from (5.3) and (5.4).

Extended spinor connections naturally arise when we describe the set of differentiations of extended tensor fields. According to the structural theorem proved in [1] each differentiation D is a sum of the three special types of differentiations:

(1) a spatial covariant differentiation;
(2) several native vertical multivariate differentiations;
(3) a degenerate differentiation.

Native vertical multivariate differentiations are most simple in the above list in that sense that they are given by the shortest formulas of all three. Let $\mathbf{S}[P]$ be a spin-tensor from the list (4.3) and let $(\alpha_P,\beta_P|\nu_P,\gamma_P|m_P,n_P)$ be its type. Assume that $\mathbf{Y}$ is some arbitrary extended spin-tensorial field of this type $(\alpha_P,\beta_P|\nu_P,\gamma_P|m_P,n_P)$. Then for $\mathbf{Y}$ the native vertical multivariate differentiation

$\boldsymbol\nabla_{\mathbf Y}[P]$ along this spin-tensorial field is defined (see [1]). In an equipped local chart U it is represented by the native multivariate derivative

$$\nabla^{j_1\ldots\,j_\beta\bar j_1\ldots\,\bar j_\gamma k_1\ldots\,k_n}_{i_1\ldots\,i_\alpha\bar i_1\ldots\,\bar i_\nu h_1\ldots\,h_m}[P]=\frac{\partial}{\partial S^{\,i_1\ldots\,i_\alpha\bar i_1\ldots\,\bar i_\nu h_1\ldots\,h_m}_{j_1\ldots\,j_\beta\bar j_1\ldots\,\bar j_\gamma k_1\ldots\,k_n}[P]},\tag{5.9}$$

where $\alpha=\alpha_P$, $\beta=\beta_P$, $\nu=\nu_P$, $\gamma=\gamma_P$, $m=m_P$, $n=n_P$. Similarly, if $\mathbf Y$ is some extended spin-tensorial field of the type $(\nu_P,\gamma_P|\alpha_P,\beta_P|m_P,n_P)$, then the barred native vertical multivariate differentiation $\bar{\boldsymbol\nabla}_{\mathbf Y}[P]$ along $\mathbf Y$ is defined (see [1] again). In an equipped local chart U this differentiation is represented by the corresponding barred native multivariate derivative

$$\bar\nabla^{j_1\ldots\,j_\gamma\,\bar j_1\ldots\,\bar j_\beta\,k_1\ldots\,k_n}_{i_1\ldots\,i_\nu\,\bar i_1\ldots\,\bar i_\alpha\,h_1\ldots\,h_m}[P]=\frac{\partial}{\overline{\partial S^{\,\bar i_1\ldots\,\bar i_\alpha\,i_1\ldots\,i_\nu\,h_1\ldots\,h_m}_{\bar j_1\ldots\,\bar j_\beta\,j_1\ldots\,j_\gamma\,k_1\ldots\,k_n}[P]}}.\tag{5.10}$$

Here again $\alpha=\alpha_P$, $\beta=\beta_P$, $\nu=\nu_P$, $\gamma=\gamma_P$, $m=m_P$, $n=n_P$. The derivatives (5.9) and (5.10) are called native because they are represented by partial derivatives with respect to the variables (4.5) native for the composite bundle (4.2). Note that they do not require and do not provide any geometric structures other than those already exist due to the bundle (4.2).

Degenerate differentiations are given by a little bit more complicated formulas. Their structure is described by the following theorem proved in [1].

Theorem 5.1. *Defining a degenerate differentiation D of extended spin-tensorial fields is equivalent to fixing three extended spin-tensorial fields $\mathfrak S$, $\bar{\mathfrak S}$, and $\mathbf S$ of the types $(1,1|0,0|0,0)$, $(0,0|1,1|0,0)$, and $(0,0|0,0|1,1)$ respectively.*

Let D be a degenerate differentiation and let $\mathbf X$ be an arbitrary smooth spin-tensorial field of the type $(\varepsilon,\eta|\sigma,\zeta|e,f)$. In an equipped local chart U the components of the field $D(\mathbf X)$ are given by the formula

$$\begin{aligned}
&DX^{a_1\ldots\,a_\varepsilon\bar a_1\ldots\,\bar a_\sigma c_1\ldots\,c_e}_{b_1\ldots\,b_\eta\bar b_1\ldots\,\bar b_\zeta d_1\ldots\,d_f}=\sum^{\varepsilon}_{\mu=1}\sum^{2}_{v_\mu=1}\mathfrak S^{a_\mu}_{v_\mu}\ X^{a_1\ldots\,v_\mu\,\ldots\,a_\varepsilon\bar a_1\ldots\,\bar a_\sigma c_1\ldots\,c_e}_{b_1\ldots\,\ldots\,\ldots\,b_\eta\bar b_1\ldots\,\bar b_\zeta d_1\ldots\,d_f}\,-\\
&\qquad-\sum^{\eta}_{\mu=1}\sum^{2}_{w_\mu=1}\mathfrak S^{w_\mu}_{b_\mu}\ X^{a_1\ldots\,\ldots\,\ldots\,a_\varepsilon\bar a_1\ldots\,\bar a_\sigma c_1\ldots\,c_e}_{b_1\ldots\,w_\mu\,\ldots\,b_\eta\bar b_1\ldots\,\bar b_\zeta d_1\ldots\,d_f}\,+\\
&+\sum^{\sigma}_{\mu=1}\sum^{2}_{v_\mu=1}\bar{\mathfrak S}^{\bar a_\mu}_{v_\mu}\ X^{a_1\ldots\,a_\varepsilon\bar a_1\ldots\,v_\mu\,\ldots\,\bar a_\sigma c_1\ldots\,c_e}_{b_1\ldots\,b_\eta\bar b_1\ldots\,\ldots\,\ldots\,\bar b_\zeta d_1\ldots\,d_f}\,-\\
&\qquad-\sum^{\zeta}_{\mu=1}\sum^{2}_{w_\mu=1}\bar{\mathfrak S}^{w_\mu}_{\bar b_\mu}\ X^{a_1\ldots\,a_\varepsilon\bar a_1\ldots\,\ldots\,\ldots\,\bar a_\sigma c_1\ldots\,c_e}_{b_1\ldots\,b_\eta\bar b_1\ldots\,w_\mu\,\ldots\,\bar b_\zeta d_1\ldots\,d_f}\,+\\
&+\sum^{e}_{\mu=1}\sum^{3}_{v_\mu=0}S^{c_\mu}_{v_\mu}\ X^{a_1\ldots\,a_\varepsilon\bar a_1\ldots\,\bar a_\sigma c_1\ldots\,v_\mu\,\ldots\,c_e}_{b_1\ldots\,b_\eta\bar b_1\ldots\,\bar b_\zeta d_1\ldots\,\ldots\,\ldots\,d_f}\,-\\
&\qquad-\sum^{f}_{\mu=1}\sum^{3}_{w_\mu=0}S^{w_\mu}_{b_\mu}\ X^{a_1\ldots\,a_\varepsilon\bar a_1\ldots\,\bar a_\sigma c_1\ldots\,\ldots\,\ldots\,c_e}_{b_1\ldots\,b_\eta\bar b_1\ldots\,\bar b_\zeta d_1\ldots\,w_\mu\,\ldots\,d_f}.
\end{aligned}\tag{5.11}$$

Note that the formula (5.11) has no derivatives at all. For this reason the differentiation D given by this formula is a degenerate differentiation. Note also that $\mathfrak{S}^{a_\mu}_{v_\mu}$, $\mathfrak{S}^{w_\mu}_{b_\mu}$, $\bar{\mathfrak{S}}^{\bar a_\mu}_{v_\mu}$, $\bar{\mathfrak{S}}^{w_\mu}_{\bar b_\mu}$, $S^{c_\mu}_{v_\mu}$, and $S^{w_\mu}_{b_\mu}$ in (5.11) are the components of the extended spin-tensorial fields declared in the theorem 5.1. Do not mix them with the components of transition matrices $\mathfrak{S}$ and S taken from (1.3) and (1.5).

Spatial covariant differentiations are most complicated of the above three types of differentiations. In an equipped local chart U they are represented by the corresponding spatial covariant derivatives:

$$
\begin{aligned}
&\nabla_j X^{a_1\dots\,a_\varepsilon\bar a_1\dots\,\bar a_\sigma c_1\dots\,c_e}_{b_1\dots\,b_\eta\bar b_1\dots\,\bar b_\zeta d_1\dots\,d_f}=\sum_{k=0}^{3}\Upsilon^k_j\,\frac{\partial X^{a_1\dots\,a_\varepsilon\bar a_1\dots\,\bar a_\sigma c_1\dots\,c_e}_{b_1\dots\,b_\eta\bar b_1\dots\,\bar b_\zeta d_1\dots\,d_f}}{\partial x^k}-\\
&-\sum^{J}_{P=1}\sum^{2}_{\substack{i_1,\,\dots,\,i_\alpha\\ j_1,\,\dots,\,j_\beta\\ \bar i_1,\,\dots,\,\bar i_\nu\\ \bar j_1,\,\dots,\,\bar j_\gamma}}\!\!\dots\sum^{2}\sum^{3}_{\substack{h_1,\,\dots,\,h_m\\ k_1,\,\dots,\,k_n}}\!\!\dots\sum^{3}\Biggl(\sum^{\alpha}_{\mu=1}\sum^{2}_{v_\mu=1}\mathrm{A}^{i_\mu}_{j\,v_\mu}\ S^{\,i_1\dots\,v_\mu\,\dots\,i_\alpha\,\bar i_1\dots\,\bar i_\nu\,h_1\dots\,h_m}_{\,j_1\dots\,\dots\,\dots\,j_\beta\,\bar j_1\dots\,\bar j_\gamma\,k_1\dots\,k_n}[P]-\\
&-\sum^{\beta}_{\mu=1}\sum^{2}_{w_\mu=1}\mathrm{A}^{w_\mu}_{j\,j_\mu}\ S^{\,i_1\dots\,\dots\,\dots\,i_\alpha\,\bar i_1\dots\,\bar i_\nu\,h_1\dots\,h_m}_{\,j_1\dots\,w_\mu\,\dots\,j_\beta\,\bar j_1\dots\,\bar j_\gamma\,k_1\dots\,k_n}[P]+\sum^{\nu}_{\mu=1}\sum^{2}_{v_\mu=1}\bar{\mathrm{A}}^{\bar i_\mu}_{j\,v_\mu}\times\\
&\times S^{\,i_1\dots\,i_\alpha\,\bar i_1\dots\,v_\mu\,\dots\,\bar i_\nu\,h_1\dots\,h_m}_{\,j_1\dots\,j_\beta\,\bar j_1\dots\,\dots\,\dots\,\bar j_\gamma\,k_1\dots\,k_n}[P]-\sum^{\gamma}_{\mu=1}\sum^{2}_{w_\mu=1}\bar{\mathrm{A}}^{w_\mu}_{j\,\bar j_\mu}\times\\
&\times S^{\,i_1\dots\,i_\alpha\,\bar i_1\dots\,\dots\,\dots\,\bar i_\nu\,h_1\dots\,h_m}_{\,j_1\dots\,j_\beta\,\bar j_1\dots\,w_\mu\,\dots\,\bar j_\gamma\,k_1\dots\,k_n}[P]+\sum^{m}_{\mu=1}\sum^{3}_{v_\mu=0}\Gamma^{h_\mu}_{j\,v_\mu}\times\\
&\times S^{\,i_1\dots\,i_\alpha\,\bar i_1\dots\,\bar i_\nu\,h_1\dots\,v_\mu\,\dots\,h_m}_{\,j_1\dots\,j_\beta\,\bar j_1\dots\,\bar j_\gamma\,k_1\dots\,\dots\,\dots\,k_n}[P]-\sum^{n}_{\mu=1}\sum^{3}_{w_\mu=0}\Gamma^{w_\mu}_{j\,k_\mu}\times\\
&\times S^{\,i_1\dots\,i_\alpha\,\bar i_1\dots\,\bar i_\nu\,h_1\dots\,\dots\,\dots\,h_m}_{\,j_1\dots\,j_\beta\,\bar j_1\dots\,\bar j_\gamma\,k_1\dots\,w_\mu\,\dots\,k_n}[P]\Biggr)\,\frac{\partial X^{a_1\dots\,a_\varepsilon\bar a_1\dots\,\bar a_\sigma c_1\dots\,c_e}_{b_1\dots\,b_\eta\bar b_1\dots\,\bar b_\zeta d_1\dots\,d_f}}{\partial S^{\,i_1\dots\,i_\alpha\,\bar i_1\dots\,\bar i_\nu\,h_1\dots\,h_m}_{\,j_1\dots\,j_\beta\,\bar j_1\dots\,\bar j_\gamma\,k_1\dots\,k_n}[P]}-\\
&-\sum^{J}_{P=1}\sum^{2}_{\substack{i_1,\,\dots,\,i_\alpha\\ j_1,\,\dots,\,j_\beta\\ \bar i_1,\,\dots,\,\bar i_\nu\\ \bar j_1,\,\dots,\,\bar j_\gamma}}\!\!\dots\sum^{2}\sum^{3}_{\substack{h_1,\,\dots,\,h_m\\ k_1,\,\dots,\,k_n}}\!\!\dots\sum^{3}\Biggl(\sum^{\nu}_{\mu=1}\sum^{2}_{v_\mu=1}\mathrm{A}^{i_\mu}_{j\,v_\mu}\ \overline{S^{\,\bar i_1\dots\,\bar i_\alpha\,i_1\dots\,v_\mu\,\dots\,i_\nu h_1\dots\,h_m}_{\,\bar j_1\dots\,\bar j_\beta\,j_1\dots\,\dots\,\dots\,j_\gamma k_1\dots\,k_n}[P]}-\\
&-\sum^{\gamma}_{\mu=1}\sum^{2}_{w_\mu=1}\mathrm{A}^{w_\mu}_{j\,j_\mu}\ \overline{S^{\,\bar i_1\dots\,\bar i_\alpha\,i_1\dots\,\dots\,\dots\,i_\nu\,h_1\dots\,h_m}_{\,\bar j_1\dots\,\bar j_\beta\,j_1\dots\,w_\mu\,\dots\,j_\gamma\,k_1\dots\,k_n}[P]}+\sum^{\alpha}_{\mu=1}\sum^{2}_{v_\mu=1}\bar{\mathrm{A}}^{\bar i_\mu}_{j\,v_\mu}\times\\
&\times\overline{S^{\,\bar i_1\dots\,v_\mu\,\dots\,\bar i_\alpha\,i_1\dots\,i_\nu\,h_1\dots\,h_m}_{\,\bar j_1\dots\,\dots\,\dots\,\bar j_\beta\,j_1\dots\,j_\gamma\,k_1\dots\,k_n}[P]}-\sum^{\beta}_{\mu=1}\sum^{2}_{w_\mu=1}\bar{\mathrm{A}}^{w_\mu}_{j\,\bar j_\mu}\times\\
&\times\overline{S^{\,\bar i_1\dots\,\dots\,\dots\,\bar i_\alpha\,i_1\dots\,i_\nu\,h_1\dots\,h_m}_{\,\bar j_1\dots\,w_\mu\,\dots\,\bar j_\beta\,j_1\dots\,j_\gamma\,k_1\dots\,k_n}[P]}+\sum^{m}_{\mu=1}\sum^{3}_{v_\mu=0}\Gamma^{h_\mu}_{j\,v_\mu}\times
\end{aligned}
\tag{5.12}
$$

$$\times\,\overline{S^{\bar i_1\dots\,\bar i_\alpha\,i_1\dots\,i_\nu\,h_1\dots\,v_\mu\,\dots\,h_m}_{\bar j_1\dots\,\bar j_\beta\,j_1\dots\,j_\gamma\,k_1\dots\,\dots\,\dots\,k_n}[P]}-\sum^{n}_{\mu=1}\sum^{3}_{w_\mu=0}\Gamma^{w_\mu}_{j\,k_\mu}\,\times$$

$$\times\,\overline{S^{\bar i_1\dots\,\bar i_\alpha\,i_1\dots\,i_\nu\,h_1\dots\,\dots\,\dots\,h_m}_{\bar j_1\dots\,\bar j_\beta\,j_1\dots\,j_\gamma\,k_1\dots\,w_\mu\,\dots\,k_n}[P]}\Biggr)\,\frac{\partial X^{a_1\dots\,a_\varepsilon\,\bar a_1\dots\,\bar a_\sigma\,c_1\dots\,c_e}_{b_1\dots\,b_\eta\,\bar b_1\dots\,\bar b_\zeta\,d_1\dots\,d_f}}{\partial\overline{S^{i_1\dots\,i_\alpha\,\bar i_1\dots\,\bar i_\nu\,h_1\dots\,h_m}_{j_1\dots\,j_\beta\,\bar j_1\dots\,\bar j_\gamma\,k_1\dots\,k_n}[P]}}\,+$$

$$+\sum^{\varepsilon}_{\mu=1}\sum^{2}_{v_\mu=1}\mathrm{A}^{a_\mu}_{j\,v_\mu}\;X^{a_1\dots\,v_\mu\,\dots\,a_\varepsilon\,\bar a_1\dots\,\bar a_\sigma\,c_1\dots\,c_e}_{b_1\dots\,\dots\,\dots\,b_\eta\,\bar b_1\dots\,\bar b_\zeta\,d_1\dots\,d_f}\,-$$

$$-\sum^{\eta}_{\mu=1}\sum^{2}_{w_\mu=1}\mathrm{A}^{w_\mu}_{j\,b_\mu}\;X^{a_1\dots\,\dots\,\dots\,a_\varepsilon\,\bar a_1\dots\,\bar a_\sigma\,c_1\dots\,c_e}_{b_1\dots\,w_\mu\,\dots\,b_\eta\,\bar b_1\dots\,\bar b_\zeta\,d_1\dots\,d_f}\,+$$

$$+\sum^{\sigma}_{\mu=1}\sum^{2}_{v_\mu=1}\bar{\mathrm{A}}^{\bar a_\mu}_{j\,v_\mu}\;X^{a_1\dots\,a_\varepsilon\,\bar a_1\dots\,v_\mu\,\dots\,\bar a_\sigma\,c_1\dots\,c_e}_{b_1\dots\,b_\eta\,\bar b_1\dots\,\dots\,\dots\,\bar b_\zeta\,d_1\dots\,d_f}\,-$$

$$-\sum^{\zeta}_{\mu=1}\sum^{2}_{w_\mu=1}\bar{\mathrm{A}}^{w_\mu}_{j\,\bar b_\mu}\;X^{a_1\dots\,a_\varepsilon\,\bar a_1\dots\,\dots\,\dots\,\bar a_\sigma\,c_1\dots\,c_e}_{b_1\dots\,b_\eta\,\bar b_1\dots\,w_\mu\,\dots\,\bar b_\zeta\,d_1\dots\,d_f}\,+$$

$$+\sum^{e}_{\mu=1}\sum^{3}_{v_\mu=0}\Gamma^{c_\mu}_{j\,v_\mu}\;X^{a_1\dots\,a_\varepsilon\,\bar a_1\dots\,\bar a_\sigma\,c_1\dots\,v_\mu\,\dots\,c_e}_{b_1\dots\,b_\eta\,\bar b_1\dots\,\bar b_\zeta\,d_1\dots\,\dots\,\dots\,d_f}\,-$$

$$-\sum^{f}_{\mu=1}\sum^{3}_{w_\mu=0}\Gamma^{w_\mu}_{j\,b_\mu}\;X^{a_1\dots\,a_\varepsilon\,\bar a_1\dots\,\bar a_\sigma\,c_1\dots\,\dots\,\dots\,c_e}_{b_1\dots\,b_\eta\,\bar b_1\dots\,\bar b_\zeta\,d_1\dots\,w_\mu\,\dots\,d_f}.$$

The quantities Υ^k_j in the first term of (5.12) are taken from the expansion (1.13). The quantities $\mathrm{A}^k_{j\,i}$, $\bar{\mathrm{A}}^k_{j\,i}$, $\Gamma^k_{j\,i}$ in (5.12) are the components of an extended spinor connection introduced in the definition 5.1.

Theorem 5.2. *Defining a spatial covariant differentiation ∇ of extended spin-tensorial fields is equivalent to defining some extended spinor connection.*

The theorem 5.2 is immediate from the formula (5.12). It was first proved in [1].

6. Commutation relationships and curvature spin-tensors.

According to the theorem 5.1 any degenerate differentiation is given by three spin-tensorial fields $\mathfrak{S}$, $\bar{\mathfrak{S}}$, and $\mathbf{S}$. Let's denote it as

$$D=D(\mathfrak{S},\bar{\mathfrak{S}},\mathbf{S}).\tag{6.1}$$

Assume that we have two degenerate differentiations of the form (6.1):

$$D_1=D(\mathfrak{S}_1,\bar{\mathfrak{S}}_1,\mathbf{S}_1),\qquad\qquad D_2=D(\mathfrak{S}_2,\bar{\mathfrak{S}}_2,\mathbf{S}_2).\tag{6.2}$$

The commutator of any two differentiations is a differentiation (see section 12 in [1]). In the present case the commutator of the degenerate differentiations D_1 and D_2 in (6.2) is a degenerate differentiation:

$$[D(\mathfrak{S}_1,\bar{\mathfrak{S}}_1,\mathbf{S}_1),\,D(\mathfrak{S}_2,\bar{\mathfrak{S}}_2,\mathbf{S}_2)]=D(\mathfrak{S}_3,\bar{\mathfrak{S}}_3,\mathbf{S}_3).\tag{6.3}$$

For the extended spin-tensorial fields $\boldsymbol{\mathfrak{S}}_3$, $\bar{\boldsymbol{\mathfrak{S}}}_3$, and $\mathbf{S}_3$ in (6.3) one easily derives:

$$\begin{aligned}
\boldsymbol{\mathfrak{S}}_3 &= C(\boldsymbol{\mathfrak{S}}_1 \otimes \boldsymbol{\mathfrak{S}}_2 - \boldsymbol{\mathfrak{S}}_2 \otimes \boldsymbol{\mathfrak{S}}_1),\\
\bar{\boldsymbol{\mathfrak{S}}}_3 &= C(\bar{\boldsymbol{\mathfrak{S}}}_1 \otimes \bar{\boldsymbol{\mathfrak{S}}}_2 - \bar{\boldsymbol{\mathfrak{S}}}_2 \otimes \bar{\boldsymbol{\mathfrak{S}}}_1),\\
\mathbf{S}_3 &= C(\mathbf{S}_1 \otimes \mathbf{S}_2 - \mathbf{S}_2 \otimes \mathbf{S}_1).
\end{aligned}$$

These formulas mean that the operator-valued fields $\boldsymbol{\mathfrak{S}}_3$, $\bar{\boldsymbol{\mathfrak{S}}}_3$, and $\mathbf{S}_3$ are calculated as pointwise commutators of the corresponding fields $\boldsymbol{\mathfrak{S}}_1$, $\bar{\boldsymbol{\mathfrak{S}}}_1$, $\mathbf{S}_1$ and $\boldsymbol{\mathfrak{S}}_2$, $\bar{\boldsymbol{\mathfrak{S}}}_2$, $\mathbf{S}_2$.

Let $\nabla_{\mathbf{X}}[P]$ be the P-th native multivariate differentiation along an extended spin-tensorial field $\mathbf{X}$. Then we have the equality

$$[\nabla_{\mathbf{X}}[P],\, D(\boldsymbol{\mathfrak{S}}, \bar{\boldsymbol{\mathfrak{S}}}, \mathbf{S})] = D(\boldsymbol{\mathfrak{R}}, \bar{\boldsymbol{\mathfrak{R}}}, \mathbf{R}), \tag{6.4}$$

where

$$\boldsymbol{\mathfrak{R}} = \nabla_{\mathbf{X}}[P]\boldsymbol{\mathfrak{S}}, \qquad \bar{\boldsymbol{\mathfrak{R}}} = \nabla_{\mathbf{X}}[P]\bar{\boldsymbol{\mathfrak{S}}}, \qquad \mathbf{R} = \nabla_{\mathbf{X}}[P]\mathbf{S}. \tag{6.5}$$

Similarly, for the P-th barred native multivariate differentiation $\bar{\nabla}_{\mathbf{X}}[P]$ along some extended spin-tensorial field $\mathbf{X}$ we have the equality

$$[\bar{\nabla}_{\mathbf{X}}[P],\, D(\boldsymbol{\mathfrak{S}}, \bar{\boldsymbol{\mathfrak{S}}}, \mathbf{S})] = D(\boldsymbol{\mathfrak{R}}, \bar{\boldsymbol{\mathfrak{R}}}, \mathbf{R}), \tag{6.6}$$

where

$$\boldsymbol{\mathfrak{R}} = \bar{\nabla}_{\mathbf{X}}[P]\boldsymbol{\mathfrak{S}}, \qquad \bar{\boldsymbol{\mathfrak{R}}} = \bar{\nabla}_{\mathbf{X}}[P]\bar{\boldsymbol{\mathfrak{S}}}, \qquad \mathbf{R} = \bar{\nabla}_{\mathbf{X}}[P]\mathbf{S}. \tag{6.7}$$

The formulas (6.4), (6.5), (6.6), and (6.7) are proved by direct calculations using some equipped local chart U.

For mutual commutators of barred and non-barred native multivariate differentiations one can easily derive the following formulas:

$$\begin{gathered}
[\nabla_{\mathbf{X}}[P],\, \nabla_{\mathbf{Y}}[Q]] = \nabla_{\mathbf{V}}[Q] - \nabla_{\mathbf{U}}[P],\\
\text{where} \qquad \mathbf{V} = \nabla_{\mathbf{X}}[P]\mathbf{Y} \qquad \text{and} \qquad \mathbf{U} = \nabla_{\mathbf{Y}}[Q]\mathbf{X};
\end{gathered} \tag{6.8}$$

$$\begin{gathered}
[\nabla_{\mathbf{X}}[P],\, \bar{\nabla}_{\mathbf{Y}}[Q]] = \bar{\nabla}_{\mathbf{V}}[Q] - \nabla_{\mathbf{U}}[P],\\
\text{where} \qquad \mathbf{V} = \nabla_{\mathbf{X}}[P]\mathbf{Y} \qquad \text{and} \qquad \mathbf{U} = \bar{\nabla}_{\mathbf{Y}}[Q]\mathbf{X};
\end{gathered} \tag{6.9}$$

$$\begin{gathered}
[\bar{\nabla}_{\mathbf{X}}[P],\, \bar{\nabla}_{\mathbf{Y}}[Q]] = \bar{\nabla}_{\mathbf{V}}[Q] - \bar{\nabla}_{\mathbf{U}}[P],\\
\text{where} \qquad \mathbf{V} = \bar{\nabla}_{\mathbf{X}}[P]\mathbf{Y} \qquad \text{and} \qquad \mathbf{U} = \bar{\nabla}_{\mathbf{Y}}[Q]\mathbf{X}.
\end{gathered} \tag{6.10}$$

These formulas (6.8), (6.9), and (6.10) are also proved by direct calculations using some equipped local chart U of M.

Now assume that we have some extended spinor connection associated with the composite spin-tensorial bundle (4.2). Then the spatial covariant differentiation ∇

is defined and we can write various commutation relationships with it:

$$
\begin{gathered}
[\nabla_{\mathbf{X}},\, D(\mathfrak{S},\bar{\mathfrak{S}},\mathbf{S})]=D(\mathfrak{R},\bar{\mathfrak{R}},\mathbf{R}),\\
\text{where}\qquad \mathfrak{R}=\nabla_{\mathbf{X}}\mathfrak{S},\qquad \bar{\mathfrak{R}}=\nabla_{\mathbf{X}}\bar{\mathfrak{S}},\qquad \mathbf{R}=\nabla_{\mathbf{X}}\mathbf{S}.
\end{gathered}
\tag{6.11}
$$

The formulas for commutators of $\nabla_{\mathbf{X}}$ with barred and non-barred multivariate differentiations are more complicated than (6.11). In the case of $\nabla_{\mathbf{Y}}[P]$ we have

$$
\begin{gathered}
[\nabla_{\mathbf{X}},\,\nabla_{\mathbf{Y}}[P]]=\nabla_{\mathbf{U}}[P]+\sum^{J}_{Q=1}\nabla_{\mathbf{U}[Q]}[Q]+\\
+\sum^{J}_{Q=1}\bar{\nabla}_{\bar{\mathbf{U}}[Q]}[Q]-\nabla_{\mathbf{V}}+D(\mathfrak{N}^{+},\bar{\mathfrak{N}}^{+},\mathbf{N}^{+}),
\end{gathered}
\tag{6.12}
$$

where $\mathbf{U}=\nabla_{\mathbf{X}}\mathbf{Y}$ and $\mathbf{V}=\nabla_{\mathbf{Y}}[P]\mathbf{X}$. As for the fields $\mathbf{U}[Q]$ and $\bar{\mathbf{U}}[Q]$ in (6.12), they are defined by the following formulas:

$$
\begin{aligned}
&\mathbf{U}[Q]=-D(\mathfrak{N}^{+},\bar{\mathfrak{N}}^{+},\mathbf{N}^{+})\,\mathbf{S}[Q],\\
&\bar{\mathbf{U}}[Q]=-D(\mathfrak{N}^{+},\bar{\mathfrak{N}}^{+},\mathbf{N}^{+})\,\tau(\mathbf{S}[Q]).
\end{aligned}
\tag{6.13}
$$

Three spin-tensorial fields $\mathfrak{N}^{+}$, $\bar{\mathfrak{N}}^{+}$, $\mathbf{N}^{+}$ determining the degenerate differentiation $D(\mathfrak{N}^{+},\bar{\mathfrak{N}}^{+},\mathbf{N}^{+})$ in (6.12) and (6.13) are introduced through three dynamic curvature spin-tensors $\mathfrak{D}^{+}[P]$, $\bar{\mathfrak{D}}^{+}[P]$, and $\mathbf{D}^{+}[P]$ respectively:

$$
\begin{aligned}
&\mathfrak{N}^{+}=\mathfrak{D}^{+}[P](\mathbf{X},\mathbf{Y})=C(\mathfrak{D}^{+}[P]\otimes\mathbf{X}\otimes\mathbf{Y}),\\
&\bar{\mathfrak{N}}^{+}=\bar{\mathfrak{D}}^{+}[P](\mathbf{X},\mathbf{Y})=C(\bar{\mathfrak{D}}^{+}[P]\otimes\mathbf{X}\otimes\mathbf{Y}),\\
&\mathbf{N}^{+}=\mathbf{D}^{+}[P](\mathbf{X},\mathbf{Y})=C(\mathbf{D}^{+}[P]\otimes\mathbf{X}\otimes\mathbf{Y}).
\end{aligned}
\tag{6.14}
$$

The dynamic curvature spin-tensors $\mathfrak{D}^{+}[P]$, $\bar{\mathfrak{D}}^{+}[P]$, and $\mathbf{D}^{+}[P]$ in (6.14) are determined by the extended spinor connection through which the covariant differentiation ∇ is defined. For the components of $\mathfrak{D}^{+}[P]$, $\bar{\mathfrak{D}}^{+}[P]$, and $\mathbf{D}^{+}[P]$ we get

$$
\begin{aligned}
&\mathfrak{D}^{+\,k\,j_1\ldots\,j_\beta\bar{j}_1\ldots\,\bar{j}_\gamma k_1\ldots\,k_n}_{\;i\,j\,i_1\ldots\,i_\alpha\bar{i}_1\ldots\,\bar{i}_\nu h_1\ldots\,h_m}[P]=-\frac{\partial\,\mathrm{A}^k_{j\,i}}{\partial S^{\,i_1\ldots\,i_\alpha\bar{i}_1\ldots\,\bar{i}_\nu h_1\ldots\,h_m}_{\,j_1\ldots\,j_\beta\bar{j}_1\ldots\,\bar{j}_\gamma k_1\ldots\,k_n}[P]},\\
&\bar{\mathfrak{D}}^{+\,k\,j_1\ldots\,j_\beta\bar{j}_1\ldots\,\bar{j}_\gamma k_1\ldots\,k_n}_{\;i\,j\,i_1\ldots\,i_\alpha\bar{i}_1\ldots\,\bar{i}_\nu h_1\ldots\,h_m}[P]=-\frac{\partial\bar{\mathrm{A}}^k_{j\,i}}{\partial S^{\,i_1\ldots\,i_\alpha\bar{i}_1\ldots\,\bar{i}_\nu h_1\ldots\,h_m}_{\,j_1\ldots\,j_\beta\bar{j}_1\ldots\,\bar{j}_\gamma k_1\ldots\,k_n}[P]},\\
&D^{+\,k\,j_1\ldots\,j_\beta\bar{j}_1\ldots\,\bar{j}_\gamma k_1\ldots\,k_n}_{\;i\,j\,i_1\ldots\,i_\alpha\bar{i}_1\ldots\,\bar{i}_\nu h_1\ldots\,h_m}[P]=-\frac{\partial\Gamma^k_{j\,i}}{\partial S^{\,i_1\ldots\,i_\alpha\bar{i}_1\ldots\,\bar{i}_\nu h_1\ldots\,h_m}_{\,j_1\ldots\,j_\beta\bar{j}_1\ldots\,\bar{j}_\gamma k_1\ldots\,k_n}[P]}.
\end{aligned}
\tag{6.15}
$$

In the case of the barred P-th native multivariate differentiation along an extended spin-tensorial field $\mathbf{Y}$ the formula (6.12) is rewritten as follows:

$$
\begin{gathered}
[\nabla_{\mathbf{X}},\,\bar{\nabla}_{\mathbf{Y}}[P]]=\bar{\nabla}_{\mathbf{U}}[P]+\sum^{J}_{Q=1}\nabla_{\mathbf{U}[Q]}[Q]+\\
+\sum^{J}_{Q=1}\bar{\nabla}_{\bar{\mathbf{U}}[Q]}[Q]-\nabla_{\mathbf{V}}+D(\mathfrak{N}^{-},\bar{\mathfrak{N}}^{-},\mathbf{N}^{-}).
\end{gathered}
\tag{6.16}
$$

Here $\mathbf{U} = \nabla_{\mathbf{X}}\mathbf{Y}$ and $\mathbf{V} = \bar{\bar{\nabla}}_{\mathbf{Y}}[P]\mathbf{X}$. As for the fields $\mathbf{U}[Q]$ and $\bar{\mathbf{U}}[Q]$ in (6.16), they are defined by the following formulas:

$$
\begin{aligned}
&\mathbf{U}[Q] = -D(\mathfrak{N}^-, \bar{\mathfrak{N}}^-, \mathbf{N}^-)\, \mathbf{S}[Q],\\
&\bar{\mathbf{U}}[Q] = -D(\mathfrak{N}^-, \bar{\mathfrak{N}}^-, \mathbf{N}^-)\, \tau(\mathbf{S}[Q]).
\end{aligned}
\tag{6.17}
$$

Three spin-tensorial fields $\mathfrak{N}^-$, $\bar{\mathfrak{N}}^-$, $\mathbf{N}^-$ determining the degenerate differentiation $D(\mathfrak{N}^-, \bar{\mathfrak{N}}^-, \mathbf{N}^-)$ in (6.16) and (6.17) are introduced through other three dynamic curvature spin-tensors $\mathfrak{D}^-[P]$, $\bar{\mathfrak{D}}^-[P]$, and $\mathbf{D}^-[P]$:

$$
\begin{aligned}
&\mathfrak{N}^- = \mathfrak{D}^-[P](\mathbf{X}, \mathbf{Y}) = C(\mathfrak{D}^-[P] \otimes \mathbf{X} \otimes \mathbf{Y}),\\
&\bar{\mathfrak{N}}^- = \bar{\mathfrak{D}}^-[P](\mathbf{X}, \mathbf{Y}) = C(\bar{\mathfrak{D}}^-[P] \otimes \mathbf{X} \otimes \mathbf{Y}),\\
&\mathbf{N}^- = \mathbf{D}^-[P](\mathbf{X}, \mathbf{Y}) = C(\mathbf{D}^-[P] \otimes \mathbf{X} \otimes \mathbf{Y}).
\end{aligned}
\tag{6.18}
$$

Like $\mathfrak{D}^+[P]$, $\bar{\mathfrak{D}}^+[P]$, and $\mathbf{D}^+[P]$ in (6.14), the dynamic curvature spin-tensors $\mathfrak{D}^-[P]$, $\bar{\mathfrak{D}}^-[P]$, and $\mathbf{D}^-[P]$ in (6.18) are determined by the extended spinor connection through which the covariant differentiation ∇ is defined. For the components of these spin-tensors we get the following expressions:

$$
\begin{aligned}
&\mathfrak{D}^{-\,k\,j_1\ldots\,j_\gamma\,\bar{j}_1\ldots\,\bar{j}_\beta\,k_1\ldots\,k_n}_{\ i\,j\,i_1\ldots\,i_\nu\,\bar{i}_1\ldots\,\bar{i}_\alpha\,h_1\ldots\,h_m}[P] = -\frac{\partial\,\mathrm{A}^k_{j\,i}}{\partial S^{\bar{i}_1\ldots\,\bar{i}_\alpha\,i_1\ldots\,i_\nu\,h_1\ldots\,h_m}_{\bar{j}_1\ldots\,\bar{j}_\beta\,j_1\ldots\,j_\gamma\,k_1\ldots\,k_n}[P]},\\
&\bar{\mathfrak{D}}^{-\,k\,j_1\ldots\,j_\gamma\,\bar{j}_1\ldots\,\bar{j}_\beta\,k_1\ldots\,k_n}_{\ i\,j\,i_1\ldots\,i_\nu\,\bar{i}_1\ldots\,\bar{i}_\alpha\,h_1\ldots\,h_m}[P] = -\frac{\partial\bar{\mathrm{A}}^k_{j\,i}}{\partial S^{\bar{i}_1\ldots\,\bar{i}_\alpha\,i_1\ldots\,i_\nu\,h_1\ldots\,h_m}_{\bar{j}_1\ldots\,\bar{j}_\beta\,j_1\ldots\,j_\gamma\,k_1\ldots\,k_n}[P]},\\
&D^{-\,k\,j_1\ldots\,j_\gamma\,\bar{j}_1\ldots\,\bar{j}_\beta\,k_1\ldots\,k_n}_{\ i\,j\,i_1\ldots\,i_\nu\,\bar{i}_1\ldots\,\bar{i}_\alpha\,h_1\ldots\,h_m}[P] = -\frac{\partial\Gamma^k_{j\,i}}{\partial S^{\bar{i}_1\ldots\,\bar{i}_\alpha\,i_1\ldots\,i_\nu\,h_1\ldots\,h_m}_{\bar{j}_1\ldots\,\bar{j}_\beta\,j_1\ldots\,j_\gamma\,k_1\ldots\,k_n}[P]}.
\end{aligned}
\tag{6.19}
$$

Using (6.15) and (6.19), we can write explicit expressions for the components of the spin-tensorial fields $\mathfrak{N}^+$, $\bar{\mathfrak{N}}^+$, $\mathbf{N}^+$, $\mathfrak{N}^-$, $\bar{\mathfrak{N}}^-$, and $\mathbf{N}^-$:

$$
\mathfrak{N}^{+\,k}_{\ i} = \sum^{3}_{j=1}\ \sum^{2}_{\substack{i_1,\,\ldots\,,\,i_\alpha\\ j_1,\,\ldots\,,\,j_\beta\\ \bar{i}_1,\,\ldots\,,\,\bar{i}_\nu\\ \bar{j}_1,\,\ldots\,,\,\bar{j}_\gamma}}\!\!\ldots\!\!\sum^{3}_{\substack{h_1,\,\ldots\,,\,h_m\\ k_1,\,\ldots\,,\,k_n}} \mathfrak{D}^{+\,k\,j_1\ldots\,j_\beta\bar{j}_1\ldots\,\bar{j}_\gamma k_1\ldots\,k_n}_{\ i\,j\,i_1\ldots\,i_\alpha\bar{i}_1\ldots\,\bar{i}_\nu h_1\ldots\,h_m}[P]\ X^j\ Y^{i_1\ldots\,i_\alpha\bar{i}_1\ldots\,\bar{i}_\nu h_1\ldots\,h_m}_{j_1\ldots\,j_\beta\bar{j}_1\ldots\,\bar{j}_\gamma k_1\ldots\,k_n},
$$

$$
\mathfrak{N}^{-\,k}_{\ i} = \sum^{3}_{j=1}\ \sum^{2}_{\substack{i_1,\,\ldots\,,\,i_\alpha\\ j_1,\,\ldots\,,\,j_\beta\\ \bar{i}_1,\,\ldots\,,\,\bar{i}_\nu\\ \bar{j}_1,\,\ldots\,,\,\bar{j}_\gamma}}\!\!\ldots\!\!\sum^{3}_{\substack{h_1,\,\ldots\,,\,h_m\\ k_1,\,\ldots\,,\,k_n}} \mathfrak{D}^{-\,k\,j_1\ldots\,j_\gamma\,\bar{j}_1\ldots\,\bar{j}_\beta\,k_1\ldots\,k_n}_{\ i\,j\,i_1\ldots\,i_\nu\,\bar{i}_1\ldots\,\bar{i}_\alpha\,h_1\ldots\,h_m}[P]\ X^j\ Y^{i_1\ldots\,i_\nu\,\bar{i}_1\ldots\,\bar{i}_\alpha\,h_1\ldots\,h_m}_{j_1\ldots\,j_\gamma\,\bar{j}_1\ldots\,\bar{j}_\beta\,k_1\ldots\,k_n}.
$$

These two formulas are coordinate representations for the first two formulas in (6.14) and (6.18). Coordinate representations for other formulas in (6.14) and (6.18) are easily written by analogy.

Again assume that we have some extended spinor connection associated with the composite spin-tensorial bundle (4.2) and suppose that ∇ is the spatial covariant differentiation defined with the use this extended connection. Then

$$[\nabla_{\mathbf{X}},\,\nabla_{\mathbf{Y}}]=\nabla_{\mathbf{U}}+\sum_{Q=1}^{J}\nabla_{\mathbf{U}[Q]}[Q]+\sum_{Q=1}^{J}\bar{\nabla}_{\bar{\mathbf{U}}[Q]}[Q]+D(\mathfrak{N},\bar{\mathfrak{N}},\mathbf{N}),\tag{6.20}$$

where $\mathbf{U}=\nabla_{\mathbf{X}}\mathbf{Y}-\nabla_{\mathbf{Y}}\mathbf{X}-\mathbf{T}(\mathbf{X},\mathbf{Y})$ and

$$\mathbf{T}(\mathbf{X},\mathbf{Y})=C(\mathbf{T}\otimes\mathbf{X}\otimes\mathbf{Y}).\tag{6.21}$$

The extended spin-tensorial field $\mathbf{T}$ of the type $(0,0|0,0|1,2)$ in (6.21) is known as the *torsion field*. The components of the torsion are given by the formula

$$T^k_{ij}=\Gamma^k_{ij}-\Gamma^k_{j\,i}-c^k_{ij},\tag{6.22}$$

where c^k_{ij} are taken from (1.14) or from (1.15). The extended spin tensorial fields $\mathbf{U}[Q]$ and $\bar{\mathbf{U}}[Q]$ in (6.20) are obtained by applying $D(\mathfrak{N},\bar{\mathfrak{N}},\mathbf{N})$ to the native fields:

$$\begin{aligned}&\mathbf{U}[Q]=-D(\mathfrak{N},\bar{\mathfrak{N}},\mathbf{N})\,\mathbf{S}[Q],\\&\bar{\mathbf{U}}[Q]=-D(\mathfrak{N},\bar{\mathfrak{N}},\mathbf{N})\,\tau(\mathbf{S}[Q]).\end{aligned}\tag{6.23}$$

The degenerate differentiation $D(\mathfrak{N},\bar{\mathfrak{N}},\mathbf{N})$ in (6.20) and (6.23) is determined by three spin-tensorial fields $\mathfrak{N}$, $\bar{\mathfrak{N}}$, $\mathbf{N}$ which are expressed through three non-dynamic curvature spin-tensors $\mathfrak{R}$, $\bar{\mathfrak{R}}$, and $\mathbf{R}$ respectively:

$$\begin{aligned}&\mathfrak{N}=\mathfrak{R}(\mathbf{X},\mathbf{Y})=C(\mathfrak{R}\otimes\mathbf{X}\otimes\mathbf{Y}),\\&\bar{\mathfrak{N}}=\bar{\mathfrak{R}}(\mathbf{X},\mathbf{Y})=C(\bar{\mathfrak{R}}\otimes\mathbf{X}\otimes\mathbf{Y}),\\&\mathbf{N}=\mathbf{R}(\mathbf{X},\mathbf{Y})=C(\mathbf{R}\otimes\mathbf{X}\otimes\mathbf{Y}).\end{aligned}\tag{6.24}$$

The components of the spin-curvature tensor $\mathfrak{R}$ in (6.24) are given by the formula

$$\begin{aligned}\mathfrak{R}^p_{qij}=&\sum_{k=0}^{3}\Upsilon^k_i\,\frac{\partial\,\mathrm{A}^p_{j\,q}}{\partial x^k}-\sum_{k=0}^{3}\Upsilon^k_j\,\frac{\partial\,\mathrm{A}^p_{i\,q}}{\partial x^k}+\sum_{h=1}^{2}\Bigl(\mathrm{A}^p_{i\,h}\;\mathrm{A}^h_{j\,q}-\mathrm{A}^p_{j\,h}\;\mathrm{A}^h_{i\,q}\Bigr)\,-\\&-\sum_{P=1}^{J}\ \sum^{2}_{\substack{i_1,\,\ldots,\,i_\alpha\\ j_1,\,\ldots,\,j_\beta\\ \bar{i}_1,\,\ldots,\,\bar{i}_\nu\\ \bar{j}_1,\,\ldots,\,\bar{j}_\gamma}}\!\!\cdots\sum^{2}\ \sum^{3}_{\substack{h_1,\,\ldots,\,h_m\\ k_1,\,\ldots,\,k_n}}\!\!\cdots\sum^{3}\left(\sum_{\mu=1}^{\alpha}\sum_{v_\mu=1}^{2}\mathrm{A}^{i_\mu}_{i\,v_\mu}\;\;S^{\,i_1\ldots\,v_\mu\,\ldots\,i_\alpha\,\bar{i}_1\ldots\,\bar{i}_\nu\,h_1\ldots\,h_m}_{\,j_1\ldots\,\ldots\,\ldots\,j_\beta\,\bar{j}_1\ldots\,\bar{j}_\gamma\,k_1\ldots\,k_n}[P]\,-\right.\\&-\sum_{\mu=1}^{\beta}\sum_{w_\mu=1}^{2}\mathrm{A}^{w_\mu}_{i\,j_\mu}\;\;S^{\,i_1\ldots\,\ldots\,\ldots\,i_\alpha\,\bar{i}_1\ldots\,\bar{i}_\nu\,h_1\ldots\,h_m}_{\,j_1\ldots\,w_\mu\,\ldots\,j_\beta\,\bar{j}_1\ldots\,\bar{j}_\gamma\,k_1\ldots\,k_n}[P]+\sum_{\mu=1}^{\nu}\sum_{v_\mu=1}^{2}\bar{\mathrm{A}}^{\bar{i}_\mu}_{i\,v_\mu}\,\times\\&\times S^{\,i_1\ldots\,i_\alpha\,\bar{i}_1\ldots\,v_\mu\,\ldots\,\bar{i}_\nu\,h_1\ldots\,h_m}_{\,j_1\ldots\,j_\beta\,\bar{j}_1\ldots\,\ldots\,\ldots\,\bar{j}_\gamma\,k_1\ldots\,k_n}[P]-\sum_{\mu=1}^{\gamma}\sum_{w_\mu=1}^{2}\bar{\mathrm{A}}^{w_\mu}_{i\,\bar{j}_\mu}\,\times\end{aligned}\tag{6.25}$$

$$\times\, S^{i_1\dots i_\alpha\,\bar i_1\dots\dots\dots\bar i_\nu\,h_1\dots h_m}_{j_1\dots j_\beta\,\bar j_1\dots w_\mu\dots\bar j_\gamma\,k_1\dots k_n}[P] + \sum_{\mu=1}^{m}\sum_{v_\mu=0}^{3}\Gamma^{h_\mu}_{i\,v_\mu}\times$$

$$\times\, S^{i_1\dots i_\alpha\,\bar i_1\dots\bar i_\nu\,h_1\dots v_\mu\dots h_m}_{j_1\dots j_\beta\,\bar j_1\dots\bar j_\gamma\,k_1\dots\dots\dots k_n}[P] - \sum_{\mu=1}^{n}\sum_{w_\mu=0}^{3}\Gamma^{w_\mu}_{i\,k_\mu}\times$$

$$\times\, S^{i_1\dots i_\alpha\,\bar i_1\dots\bar i_\nu\,h_1\dots\dots\dots h_m}_{j_1\dots j_\beta\,\bar j_1\dots\bar j_\gamma\,k_1\dots w_\mu\dots k_n}[P]\Bigg)\frac{\partial\,\mathrm{A}^{p}_{j\,q}}{\partial S^{i_1\dots i_\alpha\,\bar i_1\dots\bar i_\nu\,h_1\dots h_m}_{j_1\dots j_\beta\,\bar j_1\dots\bar j_\gamma\,k_1\dots k_n}[P]} -$$

$$-\sum_{P=1}^{J}\sum^{2}\cdots\sum^{2}_{\substack{i_1,\dots,i_\alpha\\ j_1,\dots,j_\beta\\ \bar i_1,\dots,\bar i_\nu\\ \bar j_1,\dots,\bar j_\gamma}}\sum^{3}\cdots\sum^{3}_{\substack{h_1,\dots,h_m\\ k_1,\dots,k_n}}\Bigg(\sum_{\mu=1}^{\nu}\sum_{v_\mu=1}^{2}\mathrm{A}^{i_\mu}_{i\,v_\mu}\;\overline{S^{\bar i_1\dots\bar i_\alpha\,i_1\dots v_\mu\dots i_\nu h_1\dots h_m}_{\bar j_1\dots\bar j_\beta\,j_1\dots\dots\dots j_\gamma k_1\dots k_n}[P]} -$$

$$-\sum_{\mu=1}^{\gamma}\sum_{w_\mu=1}^{2}\mathrm{A}^{w_\mu}_{i\,j_\mu}\;\overline{S^{\bar i_1\dots\bar i_\alpha\,i_1\dots\dots\dots i_\nu\,h_1\dots h_m}_{\bar j_1\dots\bar j_\beta\,j_1\dots w_\mu\dots j_\gamma\,k_1\dots k_n}[P]} + \sum_{\mu=1}^{\alpha}\sum_{v_\mu=1}^{2}\bar{\mathrm{A}}^{\bar i_\mu}_{i\,v_\mu}\times$$

$$\times\,\overline{S^{\bar i_1\dots v_\mu\dots\bar i_\alpha\,i_1\dots i_\nu\,h_1\dots h_m}_{\bar j_1\dots\dots\dots\bar j_\beta\,j_1\dots j_\gamma\,k_1\dots k_n}[P]} - \sum_{\mu=1}^{\beta}\sum_{w_\mu=1}^{2}\bar{\mathrm{A}}^{w_\mu}_{i\,\bar j_\mu}\times$$

$$\times\,\overline{S^{\bar i_1\dots\dots\dots\bar i_\alpha\,i_1\dots i_\nu\,h_1\dots h_m}_{\bar j_1\dots w_\mu\dots\bar j_\beta\,j_1\dots j_\gamma\,k_1\dots k_n}[P]} + \sum_{\mu=1}^{m}\sum_{v_\mu=0}^{3}\Gamma^{h_\mu}_{i\,v_\mu}\times$$

$$\times\,\overline{S^{\bar i_1\dots\bar i_\alpha\,i_1\dots i_\nu\,h_1\dots v_\mu\dots h_m}_{\bar j_1\dots\bar j_\beta\,j_1\dots j_\gamma\,k_1\dots\dots\dots k_n}[P]} - \sum_{\mu=1}^{n}\sum_{w_\mu=0}^{3}\Gamma^{w_\mu}_{i\,k_\mu}\times$$

$$\times\,\overline{S^{\bar i_1\dots\bar i_\alpha\,i_1\dots i_\nu\,h_1\dots\dots\dots h_m}_{\bar j_1\dots\bar j_\beta j_1\dots j_\gamma\,k_1\dots w_\mu\dots k_n}[P]}\Bigg)\frac{\partial\,\mathrm{A}^{p}_{j\,q}}{\partial\overline{S^{i_1\dots i_\alpha\,\bar i_1\dots\bar i_\nu\,h_1\dots h_m}_{j_1\dots j_\beta\,\bar j_1\dots\bar j_\gamma\,k_1\dots k_n}[P]}} +$$

$$+\sum_{P=1}^{J}\sum^{2}\cdots\sum^{2}_{\substack{i_1,\dots,i_\alpha\\ j_1,\dots,j_\beta\\ \bar i_1,\dots,\bar i_\nu\\ \bar j_1,\dots,\bar j_\gamma}}\sum^{3}\cdots\sum^{3}_{\substack{h_1,\dots,h_m\\ k_1,\dots,k_n}}\Bigg(\sum_{\mu=1}^{\alpha}\sum_{v_\mu=1}^{2}\mathrm{A}^{i_\mu}_{j\,v_\mu}\;S^{i_1\dots v_\mu\dots i_\alpha\,\bar i_1\dots\bar i_\nu\,h_1\dots h_m}_{j_1\dots\dots\dots j_\beta\,\bar j_1\dots\bar j_\gamma\,k_1\dots k_n}[P] -$$

$$-\sum_{\mu=1}^{\beta}\sum_{w_\mu=1}^{2}\mathrm{A}^{w_\mu}_{j\,j_\mu}\;S^{i_1\dots\dots\dots i_\alpha\,\bar i_1\dots\bar i_\nu\,h_1\dots h_m}_{j_1\dots w_\mu\dots j_\beta\,\bar j_1\dots\bar j_\gamma\,k_1\dots k_n}[P] + \sum_{\mu=1}^{\nu}\sum_{v_\mu=1}^{2}\bar{\mathrm{A}}^{\bar i_\mu}_{j\,v_\mu}\times$$

$$\times\, S^{i_1\dots i_\alpha\,\bar i_1\dots v_\mu\dots\bar i_\nu\,h_1\dots h_m}_{j_1\dots j_\beta\,\bar j_1\dots\dots\dots\bar j_\gamma\,k_1\dots k_n}[P] - \sum_{\mu=1}^{\gamma}\sum_{w_\mu=1}^{2}\bar{\mathrm{A}}^{w_\mu}_{j\,\bar j_\mu}\times$$

$$\times\, S^{i_1\dots i_\alpha\,\bar i_1\dots\dots\dots\bar i_\nu\,h_1\dots h_m}_{j_1\dots j_\beta\,\bar j_1\dots w_\mu\dots\bar j_\gamma\,k_1\dots k_n}[P] + \sum_{\mu=1}^{m}\sum_{v_\mu=0}^{3}\Gamma^{h_\mu}_{j\,v_\mu}\times$$

$$\times\, S^{i_1\dots i_\alpha\,\bar i_1\dots\bar i_\nu\,h_1\dots v_\mu\dots h_m}_{j_1\dots j_\beta\,\bar j_1\dots\bar j_\gamma\,k_1\dots\dots\dots k_n}[P] - \sum_{\mu=1}^{n}\sum_{w_\mu=0}^{3}\Gamma^{w_\mu}_{j\,k_\mu}\times$$

$$
\begin{aligned}
&\times\,S^{i_1\ldots\,i_\alpha\,\bar{i}_1\ldots\,\bar{i}_\nu\,h_1\ldots\ldots\ldots\,h_m}_{j_1\ldots\,j_\beta\,\bar{j}_1\ldots\,\bar{j}_\gamma\,k_1\ldots\,w_\mu\,\ldots\,k_n}[P]\biggr)\,\frac{\partial\,\mathrm{A}^p_{i\,q}}{\partial S^{i_1\ldots\,i_\alpha\,\bar{i}_1\ldots\,\bar{i}_\nu\,h_1\ldots\,h_m}_{j_1\ldots\,j_\beta\,\bar{j}_1\ldots\,\bar{j}_\gamma\,k_1\ldots\,k_n}[P]}\,+\\
&+\sum^J_{P=1}\sum^2_{\substack{i_1,\,\ldots\,,\,i_\alpha\\ j_1,\,\ldots\,,\,j_\beta\\ \bar{i}_1,\,\ldots\,,\,\bar{i}_\nu\\ \bar{j}_1,\,\ldots\,,\,\bar{j}_\gamma}}\!\!\ldots\sum^2\sum^3_{\substack{h_1,\,\ldots\,,\,h_m\\ k_1,\,\ldots\,,\,k_n}}\!\!\ldots\sum^3\biggl(\,\sum^{\nu}_{\mu=1}\sum^2_{v_\mu=1}\mathrm{A}^{i_\mu}_{j\,v_\mu}\ \overline{S^{\bar{i}_1\ldots\,\bar{i}_\alpha\,i_1\ldots\,v_\mu\,\ldots\,i_\nu\,h_1\ldots\,h_m}_{\bar{j}_1\ldots\,\bar{j}_\beta\,j_1\ldots\ldots\ldots\,j_\gamma\,k_1\ldots\,k_n}[P]}\,-\\
&-\sum^{\gamma}_{\mu=1}\sum^2_{w_\mu=1}\mathrm{A}^{w_\mu}_{j\,j_\mu}\ \overline{S^{\bar{i}_1\ldots\,\bar{i}_\alpha\,i_1\ldots\ldots\ldots\,i_\nu\,h_1\ldots\,h_m}_{\bar{j}_1\ldots\,\bar{j}_\beta\,j_1\ldots\,w_\mu\,\ldots\,j_\gamma\,k_1\ldots\,k_n}[P]}+\sum^{\alpha}_{\mu=1}\sum^2_{v_\mu=1}\bar{\mathrm{A}}^{\bar{i}_\mu}_{j\,v_\mu}\,\times\\
&\times\overline{S^{\bar{i}_1\ldots\,v_\mu\,\ldots\,\bar{i}_\alpha\,i_1\ldots\,i_\nu\,h_1\ldots\,h_m}_{\bar{j}_1\ldots\ldots\ldots\,\bar{j}_\beta\,j_1\ldots\,j_\gamma\,k_1\ldots\,k_n}[P]}-\sum^{\beta}_{\mu=1}\sum^2_{w_\mu=1}\bar{\mathrm{A}}^{w_\mu}_{j\,\bar{j}_\mu}\,\times\\
&\times\overline{S^{\bar{i}_1\ldots\ldots\ldots\,\bar{i}_\alpha\,i_1\ldots\,i_\nu\,h_1\ldots\,h_m}_{\bar{j}_1\ldots\,w_\mu\,\ldots\,\bar{j}_\beta\,j_1\ldots\,j_\gamma\,k_1\ldots\,k_n}[P]}+\sum^{m}_{\mu=1}\sum^3_{v_\mu=0}\Gamma^{h_\mu}_{j\,v_\mu}\,\times\\
&\times\overline{S^{\bar{i}_1\ldots\,\bar{i}_\alpha\,i_1\ldots\,i_\nu\,h_1\ldots\,v_\mu\,\ldots\,h_m}_{\bar{j}_1\ldots\,\bar{j}_\beta\,j_1\ldots\,j_\gamma\,k_1\ldots\ldots\ldots\,k_n}[P]}-\sum^{n}_{\mu=1}\sum^3_{w_\mu=0}\Gamma^{w_\mu}_{j\,k_\mu}\,\times\\
&\times\,\overline{S^{\bar{i}_1\ldots\,\bar{i}_\alpha\,i_1\ldots\,i_\nu\,h_1\ldots\ldots\ldots\,h_m}_{\bar{j}_1\ldots\,\bar{j}_\beta j_1\ldots\,j_\gamma\,k_1\ldots\,w_\mu\,\ldots\,k_n}[P]}\biggr)\,\frac{\partial\,\mathrm{A}^p_{i\,q}}{\overline{\partial S^{i_1\ldots\,i_\alpha\,\bar{i}_1\ldots\,\bar{i}_\nu\,h_1\ldots\,h_m}_{j_1\ldots\,j_\beta\,\bar{j}_1\ldots\,\bar{j}_\gamma\,k_1\ldots\,k_n}[P]}}-\sum^3_{k=0}c^k_{ij}\ \mathrm{A}^p_{kq}\,.
\end{aligned}
$$

For each P in the above formula (6.25) we implicitly assume that $\alpha=\alpha_P$, $\beta=\beta_P$, $\nu=\nu_P$, $\gamma=\gamma_P$, $m=m_P$, $n=n_P$. The components of $\bar{\boldsymbol{\mathfrak{R}}}$ are calculated similarly:

$$
\begin{aligned}
&\bar{\mathfrak{R}}^p_{qij}=\sum^3_{k=0}\Upsilon^k_i\,\frac{\partial\bar{\mathrm{A}}^p_{j\,q}}{\partial x^k}-\sum^3_{k=0}\Upsilon^k_j\,\frac{\partial\bar{\mathrm{A}}^p_{i\,q}}{\partial x^k}+\sum^2_{h=1}\Bigl(\bar{\mathrm{A}}^p_{i\,h}\,\bar{\mathrm{A}}^h_{j\,q}-\bar{\mathrm{A}}^p_{j\,h}\,\bar{\mathrm{A}}^h_{i\,q}\Bigr)\,-\\
&-\sum^J_{P=1}\sum^2_{\substack{i_1,\,\ldots\,,\,i_\alpha\\ j_1,\,\ldots\,,\,j_\beta\\ \bar{i}_1,\,\ldots\,,\,\bar{i}_\nu\\ \bar{j}_1,\,\ldots\,,\,\bar{j}_\gamma}}\!\!\ldots\sum^2\sum^3_{\substack{h_1,\,\ldots\,,\,h_m\\ k_1,\,\ldots\,,\,k_n}}\!\!\ldots\sum^3\biggl(\,\sum^{\alpha}_{\mu=1}\sum^2_{v_\mu=1}\mathrm{A}^{i_\mu}_{i\,v_\mu}\ S^{i_1\ldots\,v_\mu\,\ldots\,i_\alpha\,\bar{i}_1\ldots\,\bar{i}_\nu\,h_1\ldots\,h_m}_{j_1\ldots\ldots\ldots\,j_\beta\,\bar{j}_1\ldots\,\bar{j}_\gamma\,k_1\ldots\,k_n}[P]\,-\\
&-\sum^{\beta}_{\mu=1}\sum^2_{w_\mu=1}\mathrm{A}^{w_\mu}_{i\,j_\mu}\ S^{i_1\ldots\ldots\ldots\,i_\alpha\,\bar{i}_1\ldots\,\bar{i}_\nu\,h_1\ldots\,h_m}_{j_1\ldots\,w_\mu\,\ldots\,j_\beta\,\bar{j}_1\ldots\,\bar{j}_\gamma\,k_1\ldots\,k_n}[P]+\sum^{\nu}_{\mu=1}\sum^2_{v_\mu=1}\bar{\mathrm{A}}^{\bar{i}_\mu}_{i\,v_\mu}\,\times\\
&\times S^{i_1\ldots\,i_\alpha\,\bar{i}_1\ldots\,v_\mu\,\ldots\,\bar{i}_\nu\,h_1\ldots\,h_m}_{j_1\ldots\,j_\beta\,\bar{j}_1\ldots\ldots\ldots\,\bar{j}_\gamma\,k_1\ldots\,k_n}[P]-\sum^{\gamma}_{\mu=1}\sum^2_{w_\mu=1}\bar{\mathrm{A}}^{w_\mu}_{i\,\bar{j}_\mu}\,\times\\
&\times S^{i_1\ldots\,i_\alpha\,\bar{i}_1\ldots\ldots\ldots\,\bar{i}_\nu\,h_1\ldots\,h_m}_{j_1\ldots\,j_\beta\,\bar{j}_1\ldots\,w_\mu\,\ldots\,\bar{j}_\gamma\,k_1\ldots\,k_n}[P]+\sum^{m}_{\mu=1}\sum^3_{v_\mu=0}\Gamma^{h_\mu}_{i\,v_\mu}\,\times\\
&\times S^{i_1\ldots\,i_\alpha\,\bar{i}_1\ldots\,\bar{i}_\nu\,h_1\ldots\,v_\mu\,\ldots\,h_m}_{j_1\ldots\,j_\beta\,\bar{j}_1\ldots\,\bar{j}_\gamma\,k_1\ldots\ldots\ldots\,k_n}[P]-\sum^{n}_{\mu=1}\sum^3_{w_\mu=0}\Gamma^{w_\mu}_{i\,k_\mu}\,\times
\end{aligned}
\tag{6.26}
$$

$$\times\; S^{i_1\ldots i_\alpha\,\bar{i}_1\ldots\bar{i}_\nu\,h_1\ldots\ldots\ldots h_m}_{j_1\ldots j_\beta\,\bar{j}_1\ldots\bar{j}_\gamma\,k_1\ldots w_\mu\ldots k_n}[P]\Bigg)\,\frac{\partial\bar{\mathrm{A}}^{p}_{j\,q}}{\partial S^{i_1\ldots i_\alpha\,\bar{i}_1\ldots\bar{i}_\nu\,h_1\ldots h_m}_{j_1\ldots j_\beta\,\bar{j}_1\ldots\bar{j}_\gamma\,k_1\ldots k_n}[P]}-$$

$$-\sum_{P=1}^{J}\sum^{2}_{\substack{i_1,\ldots,i_\alpha\\ j_1,\ldots,j_\beta\\ \bar{i}_1,\ldots,\bar{i}_\nu\\ \bar{j}_1,\ldots,\bar{j}_\gamma}}\!\!\cdots\sum^{2}\sum^{3}_{\substack{h_1,\ldots,h_m\\ k_1,\ldots,k_n}}\!\!\cdots\sum^{3}\Bigg(\sum_{\mu=1}^{\nu}\sum_{v_\mu=1}^{2}\mathrm{A}^{i_\mu}_{i\,v_\mu}\;\overline{S^{\bar{i}_1\ldots\bar{i}_\alpha\,i_1\ldots v_\mu\ldots i_\nu h_1\ldots h_m}_{\bar{j}_1\ldots\bar{j}_\beta\,j_1\ldots\ldots\ldots j_\gamma k_1\ldots k_n}[P]}-$$

$$-\sum_{\mu=1}^{\gamma}\sum_{w_\mu=1}^{2}\mathrm{A}^{w_\mu}_{i\,j_\mu}\;\overline{S^{\bar{i}_1\ldots\bar{i}_\alpha\,i_1\ldots\ldots\ldots i_\nu\,h_1\ldots h_m}_{\bar{j}_1\ldots\bar{j}_\beta\,j_1\ldots w_\mu\ldots j_\gamma\,k_1\ldots k_n}[P]}+\sum_{\mu=1}^{\alpha}\sum_{v_\mu=1}^{2}\bar{\mathrm{A}}^{\bar{i}_\mu}_{i\,v_\mu}\times$$

$$\times\,\overline{S^{\bar{i}_1\ldots v_\mu\ldots\bar{i}_\alpha\,i_1\ldots i_\nu\,h_1\ldots h_m}_{\bar{j}_1\ldots\ldots\ldots\bar{j}_\beta\,j_1\ldots j_\gamma\,k_1\ldots k_n}[P]}-\sum_{\mu=1}^{\beta}\sum_{w_\mu=1}^{2}\bar{\mathrm{A}}^{w_\mu}_{i\,\bar{j}_\mu}\times$$

$$\times\,\overline{S^{\bar{i}_1\ldots\ldots\ldots\bar{i}_\alpha\,i_1\ldots i_\nu\,h_1\ldots h_m}_{\bar{j}_1\ldots w_\mu\ldots\bar{j}_\beta\,j_1\ldots j_\gamma\,k_1\ldots k_n}[P]}+\sum_{\mu=1}^{m}\sum_{v_\mu=0}^{3}\Gamma^{h_\mu}_{i\,v_\mu}\times$$

$$\times\,\overline{S^{\bar{i}_1\ldots\bar{i}_\alpha\,i_1\ldots i_\nu\,h_1\ldots v_\mu\ldots h_m}_{\bar{j}_1\ldots\bar{j}_\beta\,j_1\ldots j_\gamma\,k_1\ldots\ldots\ldots k_n}[P]}-\sum_{\mu=1}^{n}\sum_{w_\mu=0}^{3}\Gamma^{w_\mu}_{i\,k_\mu}\times$$

$$\times\;\overline{S^{\bar{i}_1\ldots\bar{i}_\alpha\,i_1\ldots i_\nu\,h_1\ldots\ldots\ldots h_m}_{\bar{j}_1\ldots\bar{j}_\beta j_1\ldots j_\gamma\,k_1\ldots w_\mu\ldots k_n}[P]}\Bigg)\,\frac{\partial\bar{\mathrm{A}}^{p}_{j\,q}}{\partial\overline{S^{i_1\ldots i_\alpha\,\bar{i}_1\ldots\bar{i}_\nu\,h_1\ldots h_m}_{j_1\ldots j_\beta\,\bar{j}_1\ldots\bar{j}_\gamma\,k_1\ldots k_n}[P]}}+$$

$$+\sum_{P=1}^{J}\sum^{2}_{\substack{i_1,\ldots,i_\alpha\\ j_1,\ldots,j_\beta\\ \bar{i}_1,\ldots,\bar{i}_\nu\\ \bar{j}_1,\ldots,\bar{j}_\gamma}}\!\!\cdots\sum^{2}\sum^{3}_{\substack{h_1,\ldots,h_m\\ k_1,\ldots,k_n}}\!\!\cdots\sum^{3}\Bigg(\sum_{\mu=1}^{\alpha}\sum_{v_\mu=1}^{2}\mathrm{A}^{i_\mu}_{j\,v_\mu}\;S^{i_1\ldots v_\mu\ldots i_\alpha\,\bar{i}_1\ldots\bar{i}_\nu\,h_1\ldots h_m}_{j_1\ldots\ldots\ldots j_\beta\,\bar{j}_1\ldots\bar{j}_\gamma\,k_1\ldots k_n}[P]-$$

$$-\sum_{\mu=1}^{\beta}\sum_{w_\mu=1}^{2}\mathrm{A}^{w_\mu}_{j\,j_\mu}\;S^{i_1\ldots\ldots\ldots i_\alpha\,\bar{i}_1\ldots\bar{i}_\nu\,h_1\ldots h_m}_{j_1\ldots w_\mu\ldots j_\beta\,\bar{j}_1\ldots\bar{j}_\gamma\,k_1\ldots k_n}[P]+\sum_{\mu=1}^{\nu}\sum_{v_\mu=1}^{2}\bar{\mathrm{A}}^{\bar{i}_\mu}_{j\,v_\mu}\times$$

$$\times\,S^{i_1\ldots i_\alpha\,\bar{i}_1\ldots v_\mu\ldots\bar{i}_\nu\,h_1\ldots h_m}_{j_1\ldots j_\beta\,\bar{j}_1\ldots\ldots\ldots\bar{j}_\gamma\,k_1\ldots k_n}[P]-\sum_{\mu=1}^{\gamma}\sum_{w_\mu=1}^{2}\bar{\mathrm{A}}^{w_\mu}_{j\,\bar{j}_\mu}\times$$

$$\times\,S^{i_1\ldots i_\alpha\,\bar{i}_1\ldots\ldots\ldots\bar{i}_\nu\,h_1\ldots h_m}_{j_1\ldots j_\beta\,\bar{j}_1\ldots w_\mu\ldots\bar{j}_\gamma\,k_1\ldots k_n}[P]+\sum_{\mu=1}^{m}\sum_{v_\mu=0}^{3}\Gamma^{h_\mu}_{j\,v_\mu}\times$$

$$\times\,S^{i_1\ldots i_\alpha\,\bar{i}_1\ldots\bar{i}_\nu\,h_1\ldots v_\mu\ldots h_m}_{j_1\ldots j_\beta\,\bar{j}_1\ldots\bar{j}_\gamma\,k_1\ldots\ldots\ldots k_n}[P]-\sum_{\mu=1}^{n}\sum_{w_\mu=0}^{3}\Gamma^{w_\mu}_{j\,k_\mu}\times$$

$$\times\;S^{i_1\ldots i_\alpha\,\bar{i}_1\ldots\bar{i}_\nu\,h_1\ldots\ldots\ldots h_m}_{j_1\ldots j_\beta\,\bar{j}_1\ldots\bar{j}_\gamma\,k_1\ldots w_\mu\ldots k_n}[P]\Bigg)\,\frac{\partial\bar{\mathrm{A}}^{p}_{i\,q}}{\partial S^{i_1\ldots i_\alpha\,\bar{i}_1\ldots\bar{i}_\nu\,h_1\ldots h_m}_{j_1\ldots j_\beta\,\bar{j}_1\ldots\bar{j}_\gamma\,k_1\ldots k_n}[P]}+$$

$$+\sum_{P=1}^{J}\sum^{2}_{\substack{i_1,\ldots,i_\alpha\\ j_1,\ldots,j_\beta\\ \bar{i}_1,\ldots,\bar{i}_\nu\\ \bar{j}_1,\ldots,\bar{j}_\gamma}}\!\!\cdots\sum^{2}\sum^{3}_{\substack{h_1,\ldots,h_m\\ k_1,\ldots,k_n}}\!\!\cdots\sum^{3}\Bigg(\sum_{\mu=1}^{\nu}\sum_{v_\mu=1}^{2}\mathrm{A}^{i_\mu}_{j\,v_\mu}\;\overline{S^{\bar{i}_1\ldots\bar{i}_\alpha\,i_1\ldots v_\mu\ldots i_\nu h_1\ldots h_m}_{\bar{j}_1\ldots\bar{j}_\beta\,j_1\ldots\ldots\ldots j_\gamma k_1\ldots k_n}[P]}-$$

$$-\sum_{\mu=1}^{\gamma}\sum_{w_\mu=1}^{2}\mathrm{A}^{w_\mu}_{j\,j_\mu}\ \overline{S^{\bar i_1\dots\,\bar i_\alpha\,i_1\dots\dots\dots\,i_\nu\,h_1\dots\,h_m}_{\bar j_1\dots\,\bar j_\beta\,j_1\dots\,w_\mu\,\dots\,j_\gamma\,k_1\dots\,k_n}[P]}+\sum_{\mu=1}^{\alpha}\sum_{v_\mu=1}^{2}\bar{\mathrm{A}}^{\bar i_\mu}_{j\,v_\mu}\times$$

$$\times\overline{S^{\bar i_1\dots\,v_\mu\,\dots\,\bar i_\alpha\,i_1\dots\,i_\nu\,h_1\dots\,h_m}_{\bar j_1\dots\dots\dots\,\bar j_\beta\,j_1\dots\,j_\gamma\,k_1\dots\,k_n}[P]}-\sum_{\mu=1}^{\beta}\sum_{w_\mu=1}^{2}\bar{\mathrm{A}}^{w_\mu}_{j\,\bar j_\mu}\times$$

$$\times\overline{S^{\bar i_1\dots\dots\dots\,\bar i_\alpha\,i_1\dots\,i_\nu\,h_1\dots\,h_m}_{\bar j_1\dots\,w_\mu\,\dots\,\bar j_\beta\,j_1\dots\,j_\gamma\,k_1\dots\,k_n}[P]}+\sum_{\mu=1}^{m}\sum_{v_\mu=0}^{3}\Gamma^{h_\mu}_{j\,v_\mu}\times$$

$$\times\overline{S^{\bar i_1\dots\,\bar i_\alpha\,i_1\dots\,i_\nu\,h_1\dots\,v_\mu\,\dots\,h_m}_{\bar j_1\dots\,\bar j_\beta\,j_1\dots\,j_\gamma\,k_1\dots\dots\dots\,k_n}[P]}-\sum_{\mu=1}^{n}\sum_{w_\mu=0}^{3}\Gamma^{w_\mu}_{j\,k_\mu}\times$$

$$\times\left.\overline{S^{\bar i_1\dots\,\bar i_\alpha\,i_1\dots\,i_\nu\,h_1\dots\dots\dots\,h_m}_{\bar j_1\dots\,\bar j_\beta j_1\dots\,j_\gamma\,k_1\dots\,w_\mu\,\dots\,k_n}[P]}\right)\frac{\partial\bar{\mathrm{A}}^p_{i\,q}}{\partial\overline{S^{i_1\dots\,i_\alpha\,\bar i_1\dots\,\bar i_\nu\,h_1\dots\,h_m}_{j_1\dots\,j_\beta\,\bar j_1\dots\,\bar j_\gamma\,k_1\dots\,k_n}[P]}}-\sum_{k=0}^{3}c^k_{ij}\,\bar{\mathrm{A}}^p_{kq}.$$

The components of the third curvature spin-tensor $\mathbf{R}$ are given by the formula

$$\begin{aligned}
&R^p_{qij}=\sum_{k=0}^{3}\Upsilon^k_i\,\frac{\partial\Gamma^p_{j\,q}}{\partial x^k}-\sum_{k=0}^{3}\Upsilon^k_j\,\frac{\partial\Gamma^p_{i\,q}}{\partial x^k}+\sum_{h=1}^{2}\left(\Gamma^p_{i\,h}\,\Gamma^h_{j\,q}-\Gamma^p_{j\,h}\,\Gamma^h_{i\,q}\right)-\\
&-\sum_{P=1}^{J}\sum^{2}_{\substack{i_1,\,\dots,\,i_\alpha\\ j_1,\,\dots,\,j_\beta\\ \bar i_1,\,\dots,\,\bar i_\nu\\ \bar j_1,\,\dots,\,\bar j_\gamma}}\!\!\cdots\sum^{2}\sum^{3}_{\substack{h_1,\,\dots,\,h_m\\ k_1,\,\dots,\,k_n}}\!\!\cdots\sum^{3}\left(\sum_{\mu=1}^{\alpha}\sum_{v_\mu=1}^{2}\mathrm{A}^{i_\mu}_{i\,v_\mu}\ S^{i_1\dots\,v_\mu\,\dots\,i_\alpha\,\bar i_1\dots\,\bar i_\nu\,h_1\dots\,h_m}_{j_1\dots\dots\dots\,j_\beta\,\bar j_1\dots\,\bar j_\gamma\,k_1\dots\,k_n}[P]-\right.\\
&-\sum_{\mu=1}^{\beta}\sum_{w_\mu=1}^{2}\mathrm{A}^{w_\mu}_{i\,j_\mu}\ S^{i_1\dots\dots\dots\,i_\alpha\,\bar i_1\dots\,\bar i_\nu\,h_1\dots\,h_m}_{j_1\dots\,w_\mu\,\dots\,j_\beta\,\bar j_1\dots\,\bar j_\gamma\,k_1\dots\,k_n}[P]+\sum_{\mu=1}^{\nu}\sum_{v_\mu=1}^{2}\bar{\mathrm{A}}^{\bar i_\mu}_{i\,v_\mu}\times\\
&\times S^{i_1\dots\,i_\alpha\,\bar i_1\dots\,v_\mu\,\dots\,\bar i_\nu\,h_1\dots\,h_m}_{j_1\dots\,j_\beta\,\bar j_1\dots\dots\dots\,\bar j_\gamma\,k_1\dots\,k_n}[P]-\sum_{\mu=1}^{\gamma}\sum_{w_\mu=1}^{2}\bar{\mathrm{A}}^{w_\mu}_{i\,\bar j_\mu}\times\\
&\times S^{i_1\dots\,i_\alpha\,\bar i_1\dots\dots\dots\,\bar i_\nu\,h_1\dots\,h_m}_{j_1\dots\,j_\beta\,\bar j_1\dots\,w_\mu\,\dots\,\bar j_\gamma\,k_1\dots\,k_n}[P]+\sum_{\mu=1}^{m}\sum_{v_\mu=0}^{3}\Gamma^{h_\mu}_{i\,v_\mu}\times\\
&\times S^{i_1\dots\,i_\alpha\,\bar i_1\dots\,\bar i_\nu\,h_1\dots\,v_\mu\,\dots\,h_m}_{j_1\dots\,j_\beta\,\bar j_1\dots\,\bar j_\gamma\,k_1\dots\dots\dots\,k_n}[P]-\sum_{\mu=1}^{n}\sum_{w_\mu=0}^{3}\Gamma^{w_\mu}_{i\,k_\mu}\times\\
&\times\left.S^{i_1\dots\,i_\alpha\,\bar i_1\dots\,\bar i_\nu\,h_1\dots\dots\dots\,h_m}_{j_1\dots\,j_\beta\,\bar j_1\dots\,\bar j_\gamma\,k_1\dots\,w_\mu\,\dots\,k_n}[P]\right)\frac{\partial\Gamma^p_{j\,q}}{\partial S^{i_1\dots\,i_\alpha\,\bar i_1\dots\,\bar i_\nu\,h_1\dots\,h_m}_{j_1\dots\,j_\beta\,\bar j_1\dots\,\bar j_\gamma\,k_1\dots\,k_n}[P]}-\\
&-\sum_{P=1}^{J}\sum^{2}_{\substack{i_1,\,\dots,\,i_\alpha\\ j_1,\,\dots,\,j_\beta\\ \bar i_1,\,\dots,\,\bar i_\nu\\ \bar j_1,\,\dots,\,\bar j_\gamma}}\!\!\cdots\sum^{2}\sum^{3}_{\substack{h_1,\,\dots,\,h_m\\ k_1,\,\dots,\,k_n}}\!\!\cdots\sum^{3}\left(\sum_{\mu=1}^{\nu}\sum_{v_\mu=1}^{2}\mathrm{A}^{i_\mu}_{i\,v_\mu}\ \overline{S^{\bar i_1\dots\,\bar i_\alpha\,i_1\dots\,v_\mu\,\dots\,i_\nu h_1\dots\,h_m}_{\bar j_1\dots\,\bar j_\beta\,j_1\dots\dots\dots\,j_\gamma k_1\dots\,k_n}[P]}-\right.
\end{aligned}\tag{6.27}$$

$$-\sum_{\mu=1}^{\gamma}\sum_{w_\mu=1}^{2} \mathrm{A}^{w_\mu}_{i\,j_\mu}\ \overline{S^{\bar i_1\ldots\bar i_\alpha\, i_1\ldots\ldots\ldots i_\nu\, h_1\ldots h_m}_{\bar j_1\ldots \bar j_\beta\, j_1\ldots w_\mu\ldots j_\gamma\, k_1\ldots k_n}[P]} + \sum_{\mu=1}^{\alpha}\sum_{v_\mu=1}^{2}\bar{\mathrm{A}}^{\bar i_\mu}_{i\,v_\mu}\times$$

$$\times\overline{S^{\bar i_1\ldots v_\mu\ldots\bar i_\alpha\, i_1\ldots i_\nu\, h_1\ldots h_m}_{\bar j_1\ldots\ldots\ldots\bar j_\beta\, j_1\ldots j_\gamma\, k_1\ldots k_n}[P]} - \sum_{\mu=1}^{\beta}\sum_{w_\mu=1}^{2}\bar{\mathrm{A}}^{w_\mu}_{i\,\bar j_\mu}\times$$

$$\times\overline{S^{\bar i_1\ldots\ldots\ldots\bar i_\alpha\, i_1\ldots i_\nu\, h_1\ldots h_m}_{\bar j_1\ldots w_\mu\ldots\bar j_\beta\, j_1\ldots j_\gamma\, k_1\ldots k_n}[P]} + \sum_{\mu=1}^{m}\sum_{v_\mu=0}^{3}\Gamma^{h_\mu}_{i\,v_\mu}\times$$

$$\times\overline{S^{\bar i_1\ldots\bar i_\alpha\, i_1\ldots i_\nu\, h_1\ldots v_\mu\ldots h_m}_{\bar j_1\ldots\bar j_\beta\, j_1\ldots j_\gamma\, k_1\ldots\ldots\ldots k_n}[P]} - \sum_{\mu=1}^{n}\sum_{w_\mu=0}^{3}\Gamma^{w_\mu}_{i\,k_\mu}\times$$

$$\times\ \overline{S^{\bar i_1\ldots\bar i_\alpha\, i_1\ldots i_\nu\, h_1\ldots\ldots\ldots h_m}_{\bar j_1\ldots\bar j_\beta j_1\ldots j_\gamma\, k_1\ldots w_\mu\ldots k_n}[P]}\Bigg)\ \frac{\partial\Gamma^p_{j\,q}}{\partial\overline{S^{i_1\ldots i_\alpha\,\bar i_1\ldots\bar i_\nu\, h_1\ldots h_m}_{j_1\ldots j_\beta\,\bar j_1\ldots\bar j_\gamma\, k_1\ldots k_n}[P]}}+$$

$$+\sum_{P=1}^{J}\sum_{\substack{i_1,\ldots,i_\alpha\\ j_1,\ldots,j_\beta\\ \bar i_1,\ldots,\bar i_\nu\\ \bar j_1,\ldots,\bar j_\gamma}}^{2}\!\!\cdots\sum^{2}\sum_{\substack{h_1,\ldots,h_m\\ k_1,\ldots,k_n}}^{3}\!\!\cdots\sum^{3}\Bigg(\sum_{\mu=1}^{\alpha}\sum_{v_\mu=1}^{2}\mathrm{A}^{i_\mu}_{j\,v_\mu}\ S^{i_1\ldots v_\mu\ldots i_\alpha\,\bar i_1\ldots\bar i_\nu\, h_1\ldots h_m}_{j_1\ldots\ldots\ldots j_\beta\,\bar j_1\ldots\bar j_\gamma\, k_1\ldots k_n}[P]-$$

$$-\sum_{\mu=1}^{\beta}\sum_{w_\mu=1}^{2}\mathrm{A}^{w_\mu}_{j\,j_\mu}\ S^{i_1\ldots\ldots\ldots i_\alpha\,\bar i_1\ldots\bar i_\nu\, h_1\ldots h_m}_{j_1\ldots w_\mu\ldots j_\beta\,\bar j_1\ldots\bar j_\gamma\, k_1\ldots k_n}[P]+\sum_{\mu=1}^{\nu}\sum_{v_\mu=1}^{2}\bar{\mathrm{A}}^{\bar i_\mu}_{j\,v_\mu}\times$$

$$\times S^{i_1\ldots i_\alpha\,\bar i_1\ldots v_\mu\ldots\bar i_\nu\, h_1\ldots h_m}_{j_1\ldots j_\beta\,\bar j_1\ldots\ldots\ldots\bar j_\gamma\, k_1\ldots k_n}[P]-\sum_{\mu=1}^{\gamma}\sum_{w_\mu=1}^{2}\bar{\mathrm{A}}^{w_\mu}_{j\,\bar j_\mu}\times$$

$$\times S^{i_1\ldots i_\alpha\,\bar i_1\ldots\ldots\ldots\bar i_\nu\, h_1\ldots h_m}_{j_1\ldots j_\beta\,\bar j_1\ldots w_\mu\ldots\bar j_\gamma\, k_1\ldots k_n}[P]+\sum_{\mu=1}^{m}\sum_{v_\mu=0}^{3}\Gamma^{h_\mu}_{j\,v_\mu}\times$$

$$\times S^{i_1\ldots i_\alpha\,\bar i_1\ldots\bar i_\nu\, h_1\ldots v_\mu\ldots h_m}_{j_1\ldots j_\beta\,\bar j_1\ldots\bar j_\gamma\, k_1\ldots\ldots\ldots k_n}[P]-\sum_{\mu=1}^{n}\sum_{w_\mu=0}^{3}\Gamma^{w_\mu}_{j\,k_\mu}\times$$

$$\times\ S^{i_1\ldots i_\alpha\,\bar i_1\ldots\bar i_\nu\, h_1\ldots\ldots\ldots h_m}_{j_1\ldots j_\beta\,\bar j_1\ldots\bar j_\gamma\, k_1\ldots w_\mu\ldots k_n}[P]\Bigg)\ \frac{\partial\Gamma^p_{i\,q}}{\partial S^{i_1\ldots i_\alpha\,\bar i_1\ldots\bar i_\nu\, h_1\ldots h_m}_{j_1\ldots j_\beta\,\bar j_1\ldots\bar j_\gamma\, k_1\ldots k_n}[P]}+$$

$$+\sum_{P=1}^{J}\sum_{\substack{i_1,\ldots,i_\alpha\\ j_1,\ldots,j_\beta\\ \bar i_1,\ldots,\bar i_\nu\\ \bar j_1,\ldots,\bar j_\gamma}}^{2}\!\!\cdots\sum^{2}\sum_{\substack{h_1,\ldots,h_m\\ k_1,\ldots,k_n}}^{3}\!\!\cdots\sum^{3}\Bigg(\sum_{\mu=1}^{\nu}\sum_{v_\mu=1}^{2}\mathrm{A}^{i_\mu}_{j\,v_\mu}\ \overline{S^{\bar i_1\ldots\bar i_\alpha\, i_1\ldots v_\mu\ldots i_\nu h_1\ldots h_m}_{\bar j_1\ldots\bar j_\beta\, j_1\ldots\ldots\ldots j_\gamma k_1\ldots k_n}[P]}-$$

$$-\sum_{\mu=1}^{\gamma}\sum_{w_\mu=1}^{2}\mathrm{A}^{w_\mu}_{j\,j_\mu}\ \overline{S^{\bar i_1\ldots\bar i_\alpha\, i_1\ldots\ldots\ldots i_\nu\, h_1\ldots h_m}_{\bar j_1\ldots\bar j_\beta\, j_1\ldots w_\mu\ldots j_\gamma\, k_1\ldots k_n}[P]}+\sum_{\mu=1}^{\alpha}\sum_{v_\mu=1}^{2}\bar{\mathrm{A}}^{\bar i_\mu}_{j\,v_\mu}\times$$

$$\times\overline{S^{\bar i_1\ldots v_\mu\ldots\bar i_\alpha\, i_1\ldots i_\nu\, h_1\ldots h_m}_{\bar j_1\ldots\ldots\ldots\bar j_\beta\, j_1\ldots j_\gamma\, k_1\ldots k_n}[P]}-\sum_{\mu=1}^{\beta}\sum_{w_\mu=1}^{2}\bar{\mathrm{A}}^{w_\mu}_{j\,\bar j_\mu}\times$$

$$\times\overline{S^{\bar{i}_1\ldots\ldots\ldots\bar{i}_\alpha\,i_1\ldots\,i_\nu\,h_1\ldots\,h_m}_{\bar{j}_1\ldots\,w_\mu\,\ldots\bar{j}_\beta\,j_1\ldots\,j_\gamma\,k_1\ldots\,k_n}[P]}+\sum^{m}_{\mu=1}\sum^{3}_{v_\mu=0}\Gamma^{h_\mu}_{j\,v_\mu}\times$$

$$\times\overline{S^{\bar{i}_1\ldots\bar{i}_\alpha\,i_1\ldots\,i_\nu\,h_1\ldots\,v_\mu\,\ldots\,h_m}_{\bar{j}_1\ldots\bar{j}_\beta\,j_1\ldots\,j_\gamma\,k_1\ldots\ldots\ldots\,k_n}[P]}-\sum^{n}_{\mu=1}\sum^{3}_{w_\mu=0}\Gamma^{w_\mu}_{j\,k_\mu}\times$$

$$\times\ \overline{S^{\bar{i}_1\ldots\bar{i}_\alpha\,i_1\ldots\,i_\nu\,h_1\ldots\ldots\ldots\,h_m}_{\bar{j}_1\ldots\bar{j}_\beta j_1\ldots\,j_\gamma\,k_1\ldots\,w_\mu\,\ldots\,k_n}[P]}\Biggr)\,\frac{\partial\Gamma^p_{i\,q}}{\partial\overline{S^{i_1\ldots\,i_\alpha\,\bar{i}_1\ldots\bar{i}_\nu\,h_1\ldots\,h_m}_{j_1\ldots\,j_\beta\,\bar{j}_1\ldots\bar{j}_\gamma\,k_1\ldots\,k_n}[P]}}-\sum^{3}_{k=0}c^k_{ij}\,\Gamma^p_{kq}.$$

Like in (6.25), in the above formulas (6.26) and (6.27) for each P we implicitly assume that $\alpha=\alpha_P$, $\beta=\beta_P$, $\nu=\nu_P$, $\gamma=\gamma_P$, $m=m_P$, $n=n_P$. Moreover, like in (6.22) the quantities c^k_{ij} in the last terms of (6.25), (6.26), and (6.27) are taken from (1.14) or from (1.15).

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